An introduction to magnetic reconnection

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A very short history of magnetic reconnection

- 1946 Giovanelli suggested reconnection as a mechanism for particle accleration in solar flares.
- 1957-58 Sweet and Parker developed the first quantitative model
- 1961 Dungey investigated the interaction between a dipole (magnetosphere) and the surrounding (interplanetary) magnetic field which included reconnection.
- 1964 Petschek proposed another model in order to overcome the slow reconnection rate of the Sweet-Parker model
- 1974 J B Taylor explains the turbulent relaxation of a Reversed Field Pinch by a cascade of reconnection processes which preserve only the total helicity.
- 1975 Kadomtsev explains the sawtooth crash (m=1,n=1 mode) in a tokamak by reconnection
- 1988 Schindler et al. introduced the concept of generalised magnetic reconnection in three dimensions

Literature:

- several thousands publications which refer to magnetic reconnection
- Biskamp, D., Magnetic Reconnection in Plasmas, CUP 2000
- Priest and Forbes, Magnetic Reconnection, CUP 2000
- Reconnection of Magnetic Fields, Editors: J. Birn and E. R. Priest, CUP 2007

A first impression



"Magnetic reconnection is a physical process in highly conducting plasmas in which the magnetic topology is rearranged and magnetic energy is converted to kinetic energy, thermal energy, and particle acceleration. Magnetic reconnection occurs on timescales intermediate between slow resistive diffusion of the magnetic field and fast Alfvénic timescales"

(from wikipedia)

This illustrates a 2D stationary process, magnetic field of x-type structure, plasma flow (yellow arrows) is of x-type as well, jxB forces can drive plasma

.... which is misleading in many aspects ...

Occurrence of magnetic reconnection:

• Fusion devices, in particular Tokamaks. Sawtooth oscillations limit confinement (see e.g Kadomtsev, 1987).



Reconnection Experiments: Swarthmore Spheromak Experiment (SSX), Princeton Magnetic Reconnection Experiment (MRX), High Temperature Plasma Center Tokyo (TS-3/4), Versatile Toroidal Facility (VTF) at MIT, ...

Magnetospheres of planets





Coronal Mass Ejection observed by SDO

- Solar Flares, Coronal Mass Ejections
- Accretion disks around black holes
- Magnetars (Soft Gamma Repeaters)

Why is reconnection important?

Process which is capable of releasing large amounts of magnetic energy in a comparatively short time in a plasma with a high magnetic Reynolds number.

Reconnection in other systems

•Hydrodynamics: Reconnection of vorticity ("crosslinking " or "cut and connect" of vortex tubes), e.g. [Kida, S., and M. Takaoka, Vortex Reconnection, Annu. Rev. Fluid Mech. 26, 169 (1994)]

•Superfluids: Reconnection of quantized vortex elements [e.g. Koplik, J. and Levine, H., Vortex reconnection in superfluid helium, Phys. Rev. Letters 71, 1375, (1993)]

Reconnection in other systems

- •Cosmic Strings: Reconnection of topological defects [e.g. Shellard, E.P.S., Cosmic String interactions, Nucl. Phys. B 282, 624, (1987)]
- Liquid Crystals: Reconnection of topological defects [e.g. Chuang, I., Durrer, R., Turok, N., and Yurke, B., Cosmology in the laboratory: Defect Dynamics in Liquid Crystals, Science 251, 1336, (1991)]
- •Knot theory: Surgery of framed knots [e.g. Kauffman, L.H. (1991)Knots and Physics, World Scientific, London]
- Enzymology: Reconnection (also "recombination") of strands of the DNA
- [e.g. Sumners D., Untangling DNA, Math. Intell. 12, 71-80 (1990)]



What have these systems in common?

- A primarily ideal, i.e. topology conserving dynamics.
- A (spontaneous) local non-ideal process, which changes the topology.

Characteristics of magnetic reconnection:

- a) it generates an electric field which accelerates particles parallel to B
- b) it dissipates magnetic energy (direct heating)
- c) it accelerates plasma, i.e. converts magnetic energy into kinetic energy
- d) it changes the magnetic topology (further relaxation can release more energy)

a)-c) are not properties of magnetic reconnection alone, but occur also in other plasma processes. We therefore concentrate here on aspect d).



A very crude definition of magnetic reconnection:

Magnetic reconnection is a process by which a magnetic field in an almost ideal plasma changes its topology.

almost ideal plasma: a plasma which satisfies $E + w \times B = 0$ almost everywhere in the domain under consideration

topology of magnetic flux: The connectivity of magnetic field lines (flux tubes) within the domain or between boundaries of the domain



Ideal evolution:

 $\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0 \quad \Rightarrow \quad \frac{\partial}{\partial t} \mathbf{B} - \nabla \times \mathbf{v} \times \mathbf{B} = 0 \quad \Rightarrow \quad \int_{C^2(t)} \mathbf{B} \cdot d\mathbf{a} = const. \text{ for a comoving surface } C^2,$ Alfvén's Theorem (1942): The flux through any comoving surface is conserved.

Mathematically this is an application of the Lie-derivative theorem: The magnetic field is transported (or Lie-dragged, Lie-transported, advected) by the flow v.

- \Rightarrow Topology conservation
- \Rightarrow Conservation of flux,
- \Rightarrow Conservation of field lines
- \Rightarrow Conservation of null points
- ⇒ Conservation of knots and linkages of field lines

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Role of ideal evolution in reconnection:

- Storage of magnetic energy
- Generates small scales (typically current sheets)



Role of ideal evolution in reconnection:

Under ideal evolution a non-trivial topology can store energy



non-trivial topology: (linkage of magnetic flux) ideal relaxation cannot reduce the magnetic energy to zero.

Ideal relaxation of a non-trivial magnetic field often leads to current sheets

Non-ideal evolution:

• Reconnection can only occur if the plasma dynamics is non-ideal.

 $\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{N} \quad (\text{e.g.} \quad \mathbf{N} = \eta \mathbf{j}) \quad \Rightarrow \frac{\partial}{\partial t} \mathbf{B} - \nabla \times \mathbf{v} \times \mathbf{B} = \nabla \times \mathbf{N}$ Ohms law induction equation

• For
$$\mathbf{N} = \delta \mathbf{v} \times \mathbf{B} + \nabla \Phi$$
 $\Rightarrow \frac{\partial}{\partial t} \mathbf{B} - \nabla \times \mathbf{w} \times \mathbf{B} = 0$ for $\mathbf{w} = \mathbf{v} - \delta \mathbf{v}$

the evolution is still ideal w.r.t. the velocity w (flux transport velocity). Flux transport velocities are non-unique. The difference v - w is called slippage (of plasma w.r.t. the field)

E.g Hall MHD:
$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \frac{1}{e \ n} \mathbf{j} \times \mathbf{B} \Rightarrow \mathbf{w} = \mathbf{v}_e = \mathbf{v} - \frac{1}{e \ n} \mathbf{j}$$

• For $N \neq \delta v \times B + \nabla \Phi$ the topology of the field changes. If this occurs in an isolated region only it is called reconnection. If it occurs globally it is called diffusion.

Reconnection:

 $\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{N}$ with $\mathbf{N} \neq \delta \mathbf{v} \times \mathbf{B} + \nabla \Phi$ implies either

a) $E \cdot B \neq 0$ This is 3D- (or $B \neq 0$ or $E \cdot B \neq 0$) reconnection or 2.5D (guide field) reconnection

b) $E \cdot B = 0$ and $E \neq 0$ where B = 0. This is 2D (or x-point or $E \cdot B = 0$) reconnection.

Remark: Note that these are conditions on the evolution of the electromagnetic field - they are not restricted to MHD.

Two cases of reconnection: vanishing or non-vanishing helicity source term $(E \cdot B)$.

Reconnection geometry:

• Diffusion region: domain where INI >> Iv x BI



2D: B(x,y) and v(x,y) are in one plane. The diffusion region (gray area) is bounded in the plane. 2.5D: B and v are functions of (x,y) but B has a component out of the plane. The diffusion region is bounded in (x,y) but not in z.



3D: B and v are functions of (x,y,z).Also the diffusion region is bounded in all 3 dimensions

Historically most work on reconnection has been done using the (much more simple) 2D and 2.5D case. However, while this approximation is justified for Tokamaks and other technical plasmas which have an inbuilt symmetry, in astrophysical plasmas reconnection is inherently 3D.

Magnetic Helicity:

The homogeneous Maxwell's equation yield a balance equation for the helicity density:

$$\frac{\partial}{\partial t} \underbrace{\mathbf{A} \cdot \mathbf{B}}_{\text{hel. density}} + \nabla \cdot (\underbrace{\Phi \mathbf{B} + \mathbf{E} \times \mathbf{A}}_{\text{hel. current}}) = \underbrace{-2 \mathbf{E} \cdot \mathbf{B}}_{\text{hel. source}}$$

Integrating over a closed volume (no helicity current across the boundary) yields an expression for the total helicity:

$$\frac{d}{dt} \int_{V} \mathbf{A} \cdot \mathbf{B} \ d^{3}x = -2 \int_{V} \mathbf{E} \cdot \mathbf{B} \ d^{3}x$$

The helicity integral (total helicity) is a measure for the linkage of magnetic flux in the domain



E B = 0 Reconnection:

If $E \cdot B = 0$ then N is perpendicular to B, N = $\delta v \times B$ but now δv is singular at B=0.

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{N}; \quad \mathbf{N} = \delta \mathbf{v} \times \mathbf{B}$$

$$\Rightarrow \mathbf{E} + \mathbf{w} \times \mathbf{B} = 0 \quad \text{with} \quad \mathbf{w} = \mathbf{v} - \delta \mathbf{v}$$

$$\mathbf{w} = \frac{\mathbf{E} \times \mathbf{B}}{B^2}, \quad \Rightarrow \mathbf{E} \cdot \mathbf{B} = 0; \quad \mathbf{E} \cdot \mathbf{w} = 0$$

such that the fields B and w are locally tangential to a plane perpendicular to E. Hence the evolution is (locally) two-dimensional.

- •The reconnection occurs at a null-point (x-point) of the magnetic field.
- •The source term of the magnetic helicity density (E B) vanishes.
- •The flux transport velocity w becomes singular at the x-point.



lines: magnetic field; arrows: flux transport velocity electric field perp. to plane

Note if j II E \Rightarrow j x B II v. The Lorentz force can drive the process.

E_B =0 Reconnection (continued):

Since for a generic null B is linear in x, w is of the type $-x/x^2$ along the inflow direction, hence it has a 1/x singularity at the origin.

Thus a cross-section of magnetic flux is transported in a finite time onto the null-line.

$$T = \int_0^T dt = \int_{x_0}^0 dt/dx \, dx = \int_{x_0}^0 1/w_x dx \sim x_0^2/2$$







Rate of reconnected flux:

$$\frac{d\Phi_{rec}}{dt} = \frac{d}{dt} \int \mathbf{B} \cdot \mathbf{n} da = \int E_z \, dz \quad (= \int w_x B_y \, dz)$$

Remark: The above analysis holds also for an 0-point





Rate of flux destruction/generation:

$$\frac{d}{dt}\int \mathbf{B} \cdot \mathbf{n} da = \int E_z \, dz$$

In 2D an electric field at an o-point measures the rate of destruction/generation of magnetic flux while an electric field at an x-point measures reconnection of magnetic flux.

E B = 0 Reconnection (continued):

First simple models of two-dimensional steady state reconnection have been set up by Sweet and Parker (1957/8) and later Petcheck (1964) on the basis of conservation of mass and flux in the framework of resistive MHD.

They provide a scaling of the Alfvén Mach number (the ratio of inflow plasma velocity to inflow Alfvén velocity) in terms of the Lundquist number.



 L_e is the global scale length, $v_{Ae} = B_e/\sqrt{\rho\mu_0}$ is the Alfvén speed in the inflow region

$$M_{Ae} = \frac{v_e}{v_{Ae}} = \frac{E}{v_{Ae}B_e} = \frac{\text{absolute reconnection rate}}{\text{maximum inflow of flux}} = \text{relative reconnection rate},$$

• The electric field at the x-point is an absolute measure of the reconnected flux.

$$\frac{d^2 \Phi_{\rm rec}}{dt \ dz} = E_z$$

• Both definitions are equivalent if the magnetic field does not vary along the inflow channel.

E B = 0 Reconnection (continued):

The models provide a scaling of the Alfvén Mach number in terms of the Lundquist number:

 $S = \mu_0 L_e v_{Ae} / \eta$

$$M_{Ae} = \frac{v_e}{v_{Ae}} = \frac{E}{v_{Ae}B_e} = \begin{cases} \frac{1}{\sqrt{S}} & \text{``Sweet-Parker''} & \stackrel{S=10^8}{=} 10^{-4} \\ \frac{\pi}{8\ln S} & \text{``Petschek''} & \stackrel{S=10^8}{=} 2 \cdot 10^{-2} \end{cases}$$

Due to the high Lundquist numbers in the Solar Corona (10⁸- 10¹²) the scaling of the reconnection rate with S is important.

Reconnection rates of the order of $M_{Ae} = 0.1$ yield time scales for global energy release and magnetic reconfiguration that are consistent with those seen in solar flares and magnetospheric substorms (Miller et al., 1997).

In this respect Petschek's model was a significant improvement to Sweet & Parker as it provides higher reconnection rates.

The term "fast reconnection" is used to describe reconnection rates which scale like Petschek reconnection or faster.

E_B =0 Reconnection (continued):

Warning: Many models of reconnection use the assumption that the process is timeindependent (stationary) and the electric field is replaced by a gradient (E=- $\nabla \Phi$). These are approximations which hold only for a region close to the reconnection site and for a certain time interval. Globally reconnection is always time-dependent and E \neq - $\nabla \Phi$.



globally reconnection is time-dependent



for a certain time and close to the x-point it is approximately stationary

E B = 0 Reconnection: Collisionless

Collisionless: There are not enough collisions between electrons and ions to explain the reconnective electric field at the x-point with ηj (E $\neq \eta j$).

Generalised Ohm's law:

$$\frac{m_e}{n \ e^2} \left(\frac{\partial \mathbf{j}}{\partial t} + \nabla \cdot (\mathbf{j} \ \mathbf{v} + \mathbf{v} \ \mathbf{j} - \frac{1}{n \ e} \mathbf{j} \mathbf{j} \right) = \mathbf{E} + \mathbf{v} \times \mathbf{B} - \frac{1}{n \ e} \mathbf{j} \times \mathbf{B} + \frac{1}{n \ e} \nabla \cdot \mathbf{P}_e - \eta \mathbf{j}$$



Electron inertial length: $d_e=c/\omega_{pe}$

$$[\omega_{pe} = \sqrt{4\pi n e^2/m_e}]$$

Schematic of the multiscale structure of the dissipation region during antiparallel reconnection. Electron (ion) dissipation region in white (gray) with scale size c/ω_{pe} (c/ω_{pi}). Electron (ion) flows in long (short) dashed lines. Inplane currents marked with solid dark lines and associated out-of-plane magnetic quadrupole field in gray (From Drake & Shay, Collisionless Reconnection in ``Reconnection of magnetic fields'', CUP 2007)

E B = 0 Reconnection: Collisionless



Data from a PIC simulation of anti-parallel reconnection with $m_i/m_e = 100$, $T_i/T_p = 12.0$ and c = 20.0 showing:

(a) the current J_z and in-plane magnetic

field lines;

(b)the self-generated out-of-plane field B_z

- (c) the ion outflow velocity v_x ;
- (d) the electron outflow velocity v_{xe} ; and
- (e) the Hall electric field E_y .

Noticeable are the distinct spatial scales of the electron and ion motion, the substantial value of B_z which is the signature of the standing whistler and the strong Hall electric field E_y, which maps the magnetic separatrix. The overall reconnection geometry reflects the open outflow model of Petschek rather than the elongated current layers of Sweet-Parker (From Drake & Shay, Collisionless Reconnection in ``Reconnection of magnetic fields'', CUP 2006).

E B = 0 Reconnection: Summary

- E · B =0 (2D) reconnection occurs at x-points only
- The reconnection rate (rate of reconnected flux/time/length) is given by the electric field at the x-point
- In simple stationary models of the process (Sweet-Parker / Petcheck model) the rate of reconnected flux is proportional to the (upstream) Alfvén Mach number (traditionally also called reconnection rate), which in turn can be expressed as a function of the Lundquist number.
- More detailed plasma physics (Hall MHD, two fluid, kinetic) shows a more complicated structure of the reconnection region but agrees on large scales with MHD results.

E B ≠0 Reconnection: Stationary

 $E + v \times B = N$; $E \cdot B = N \cdot B \neq 0$; stationary: $E = -\nabla \Phi$.

$$\phi = \underbrace{\int \mathbf{N}_{\parallel} ds}_{\phi_{n-id}} + \underbrace{\phi(x_0, y_0)}_{\phi_{id}} \Rightarrow \begin{cases} -\nabla \phi_{n-id.} + \mathbf{v}_{n-id.} \times \mathbf{B} = \mathbf{N} \\ -\nabla \phi_{id.} + \mathbf{v}_{id.} \times \mathbf{B} = 0 \end{cases}$$

 $\mathbf{v} = \mathbf{v}_{n-id.} + \mathbf{v}_{id.}$

General property: No direct coupling between external flow (M_{Ae}) and absolute reconnection rate.

Non-ideal solutions shows counter rotating flows above and below the reconnection region.

Basic non-ideal solution is inherent 3D and nonperiodic \Rightarrow 3D is not a simple approximation of any 2/2.5 D solution.



E ⋅ B ≠0 Reconnection: Reconnection rate



$$0 = \oint \mathbf{E} \cdot d\mathbf{l} = \int_{L} \mathbf{E} \cdot d\mathbf{l} + \int_{R_{1}} E_{r} dr - \int_{R_{2}} E_{r} dr$$
$$\Rightarrow \int_{L} \mathbf{E} \cdot d\mathbf{l} = 2 \int E_{r} dr = 2 \int v_{\phi} B_{z} dr$$

The rate of 'mismatching' of flux is given by the difference of the plasma velocity above and below the reconnection region.

$$\frac{d\Phi_{mag}}{dt} = \int_{L} \mathbf{E} \cdot d\mathbf{l} = \int_{R} \left(\mathbf{w}^{in} - \mathbf{w}^{out} \right) \times \mathbf{B} \cdot d\mathbf{r}$$

E ⋅ B ≠0 Reconnection: Time-dependent



E ⋅ B ≠0 Reconnection

There are no pairwise reconnecting field lines. For processes bounded in time reconnecting flux tubes exist which show a twist consistent with the helicity production.



E ⋅ B ≠0 Reconnection: Null points

Process depends on the direction of the electric field at the null point. Two basic cases:

- Electric field aligned with the spine axis
- Electric field tangent to the fan plane



Generic (hyperbolic) 3D null point. Fan: blue, Spine: red.



Fan planes of two null points intersect in a separator.

E ⋅ B ≠0 Reconnection: Null points

Spine aligned electric field (electric current)



Reconnection decouples the magnetic flux entering the diffusion region along the spine from the flux leaving the diffusion region along the fan plane.

The relative motion of the two fluxes (the reconnection rate) is given by the integral of the electric field along the spine.

E ⋅ B ≠0 Reconnection: Null points

Spine aligned electric field (electric current)



Reconnection decouples the magnetic flux entering the diffusion region along the spine from the flux leaving the diffusion region along the fan plane.

The relative motion of the two fluxes (the reconnection rate) is given by the integral of the electric field along the spine.

E ⋅ B ≠0 Reconnection: Turbulence

Formation of a turbulent cascade of reconnection in a braided magnetic field:



Antonia Wilmot-Smith, Gunnar Hornig, David Pontin, Cambridge Workshop on Reconnection, Fairbanks Alaska 2009

E B ≠0 Reconnection: Summary

•Three-dimensional reconnection is structurally different from 2 or 2.5 dimensional reconnection.

•The reconnection rate is given by the integral over the parallel electric field along a certain field line.

•The reconnection rate is not directly related to inflow or outflow velocities.

•The flux undergoing reconnection is restricted to thin flux layers intersecting in the non-ideal region.

•In particular we have no 1-1 correspondence of reconnecting field lines. Instead we have flux surfaces which are mapped onto each other.

•For a reconnection process which is limited in time, these flux surfaces are closed, i.e. they form perfectly reconnecting flux tubes. The counter rotating flows induce a twist in these tubes, which is consistent with the helicity production of the process.

A first impression



This illustrates a 2D stationary process, magnetic field of x-type structure, plasma flow (yellow arrows) is of x-type as well, jxB forces can drive plasma

.... which is misleading in many aspects ...

- Cartoon is 2D while almost all real reconnection processes are 3D.
- It suggests a stationary process but actually reconnection is rarely stationary.
- The cartoon suggest that a magnetic null is necessary for magnetic reconnection, which is not true.

Common Misconceptions about Reconnection:

- "Magnetic reconnection heats the plasma" The Ohmic heating due to the reconnection event itself is small due to the small size of dissipation region and a low resistivity. The major part of the magnetic energy released is converted into kinetic energy. The heating usually occurs away from the reconnection region proper via a number of non-ideal plasma processes (shocks, waves, adiabatic heating, viscous heating).
- "Magnetic reconnection occurs at an X-line" The notion of an X-line (a magnetic field line which has in a plane perpendicular to it an X-type field) is a notion deduced from 2D (or 2.5D) models. There is no distinguished "X-line" in a generic 3D magnetic field, instead there are whole regions of magnetic flux which satisfy this criterium. Moreover reconnection can also occur at an "0-line".
- "Magnetic reconnection occurs along a separator"
 A separator is a field line formed by the intersection the fan planes of two null points. It is, for instance, not present in Tokamaks but still we have reconnection in these devices.

Common Misconceptions about Reconnection:

 "Magnetic reconnection is associated with fast (Alfvénic) flows" Mach numbers of 0.02 - 0.2 are typically found in stationary 2D simulations. However, it is known that reconnection in reality is 3D and time-dependent. There is no necessity for time-dependent processes to have high flow velocities.

Some open questions:

- How does collisionless reconnection work in 3D configurations?
- Is there a "generic" dissipation mechanism?
- How common is magnetic reconnection? What is the spectrum of reconnection?
- How does reconnection work in relativistic systems?
- Observational consequences?

What is the problem?

 Complex three-dimensional geometry, complex micro physics in collisonless plasmas

- Coupling of micro (kinetic) physics and large scale dynamics (cross-scale coupling)
- Spatial resolution in simulations is too low by many orders of magnitude.

 $\frac{\text{global length scale}}{\text{electron-inertial length}} = \frac{10^8 m}{10^{-1} m} = 10^9 \qquad \frac{\text{global length scale}}{\text{gyro effects}} = \frac{10^8 m}{10^{-2} m} = 10^{10}$

(Typical resolution of a 3D simulation: 300)