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Solar Interior and Helioseismology

Professor Bill Chaplin, School of Physics & Astronomy
University of Birmingham, UK

STFC Introductory Summer School, 2017 September 11

The Unseen Interior

Sir Arthur Eddington

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The Unseen Interior

"At first sight it would seem that the deep interior of the sun and stars is less accessible to scientific investigation than any other region of the universe. Our telescopes may probe farther and farther into the depths of space; but how can we ever obtain certain knowledge of that which is hidden beneath substantial barriers? What appliance can pierce through the outer layers of a star and test the conditions within?"

A. S. Eddington, 'The Internal Constitution of the Stars', 1926, Cambridge Uni. Press, p. 1

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Pulsation opens a window!

“Ordinary stars must be viewed respectfully like objects in glass cases in museums; our fingers are itching to pinch them and test their resilience. Pulsating stars are like those fascinating models in the Science Museum provided with a button which can be pressed to set the machinery in motion. To be able to see the machinery of a star throbbing with activity is most instructive for the development of our knowledge.”

A. S. Eddington, ‘Stars and Atoms’, 1927, Oxford Uni. Press, p. 89

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NSO Workshop #27

FIFTY YEARS OF SEISMOLOGY OF THE SUN & STARS

May 6-10, 2013 • Tucson, AZ, USA

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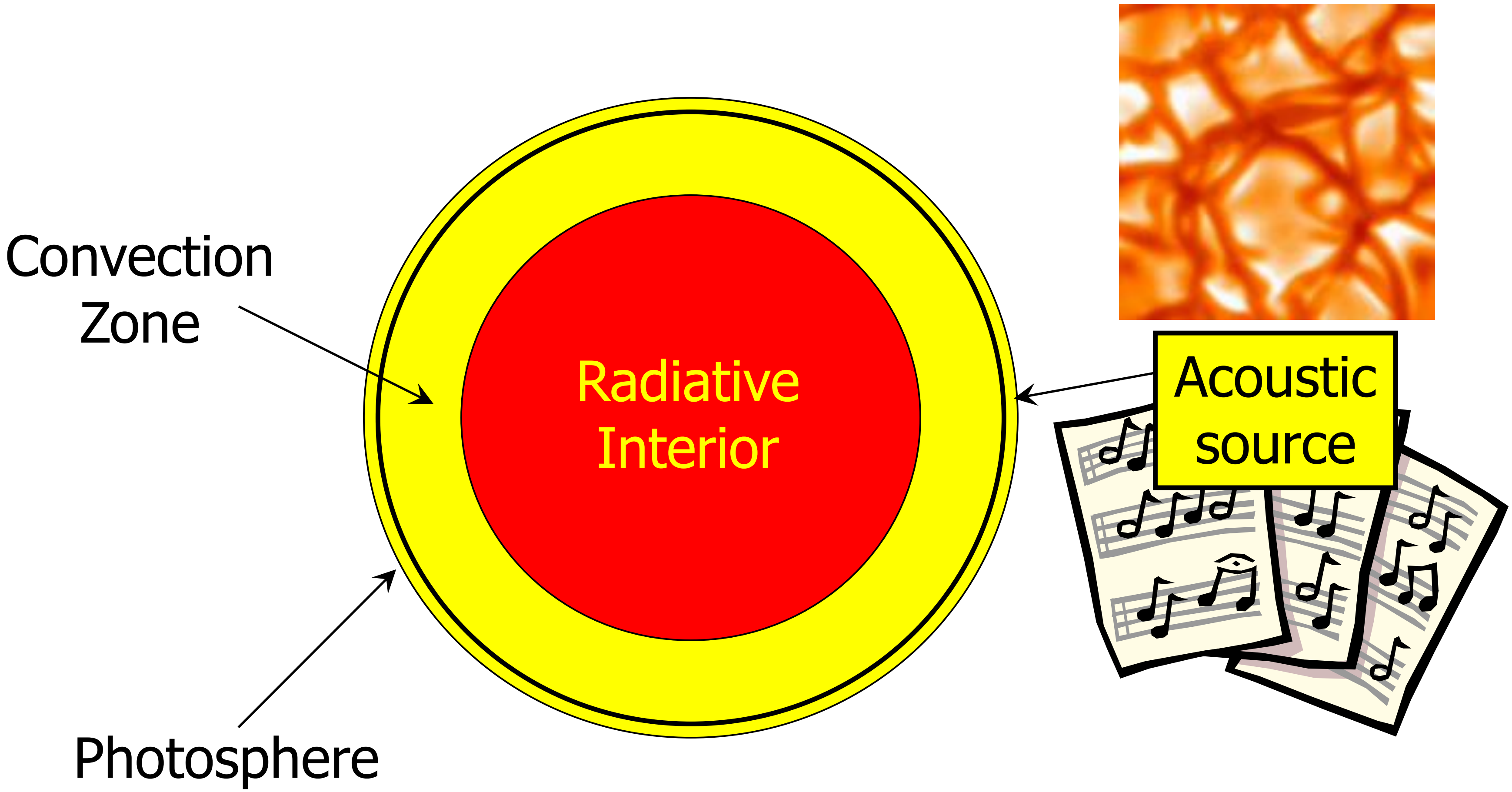
Overview

- What are resonant oscillations of the Sun?
- How do we observe the oscillations?
- What can we learn from study of the oscillations?
  - Global helioseismology
  - Local helioseismology

Sound waves generated at top of Convection Zone...



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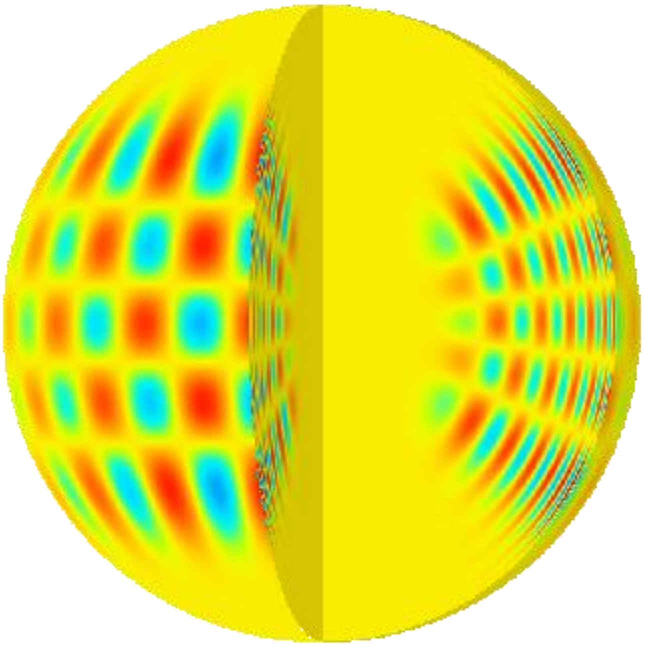
The Resonant Sun

The Sun resonates like a musical instrument...



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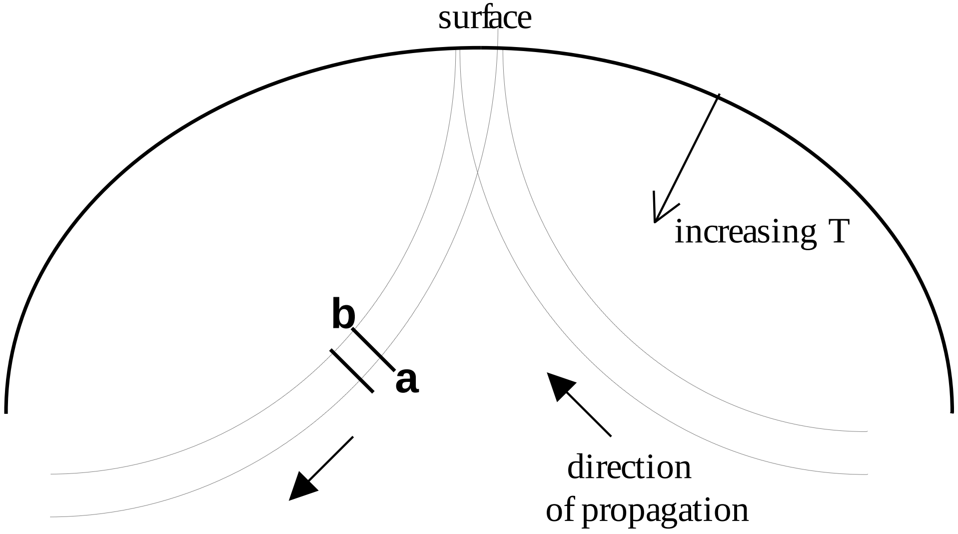
Refraction of inward-travelling waves

Waves refract if launched inward at angle to radial direction:

$$c \propto T^{1/2}$$



[*c*: sound speed; *T*: temperature]

End **a** of wave front at higher temp than **b**; so *c* there is higher!

Dispersion relation

- Simple $\omega = ck$ relation modified:
  - Interior stratified under gravity
  - Total internal reflection implies existence of cut-off frequency
  - Radial (*r*) and horizontal (*h*) wave numbers required

Dispersion relation

- Allow for different types of internal wave:
  - Acoustic waves: compression dominates
  - Buoyancy waves: displacement dominates

Dispersion relation

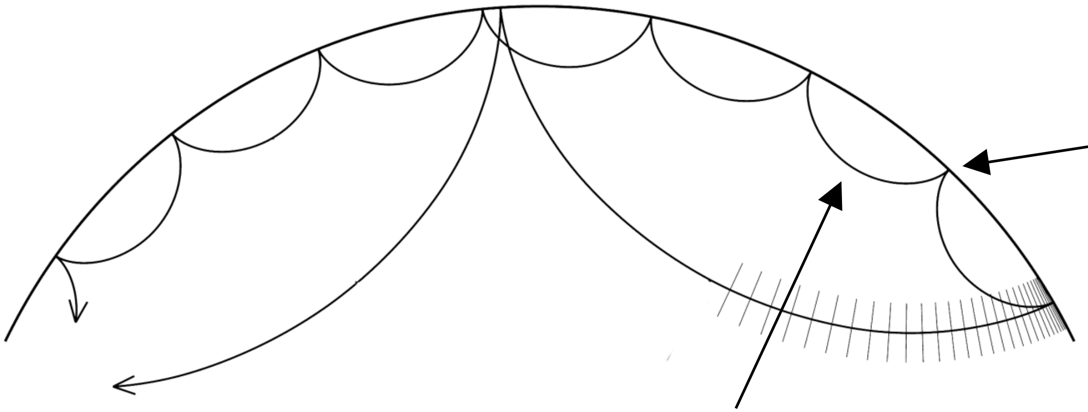
- Simple $\omega = ck$ relation modified to:

$$k_r^2 = \frac{\omega^2 - \omega_{\rm ac}^2}{c^2} + \frac{k_h^2 (N^2 - \omega^2)}{\omega^2}$$

$\omega_{\rm ac:}$ acoustic cut-off frequency

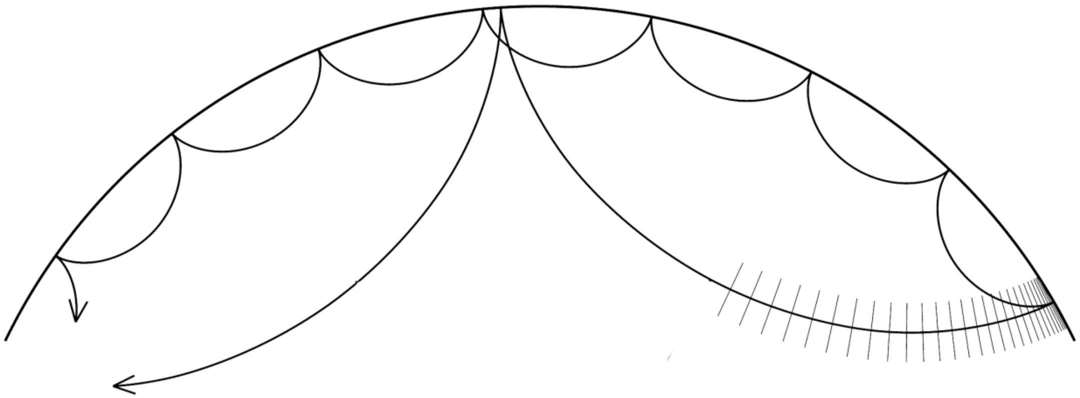
N: Brunt Väisälä frequency (characterises oscillation of fluid element displaced from rest position)

Trajectories of acoustic waves in interior



Courtesy J. Christensen-Dalsgaard

Trajectories of acoustic waves in interior



Waves launched at steeper angle to radial direction penetrate more deeply!

Courtesy J. Christensen-Dalsgaard

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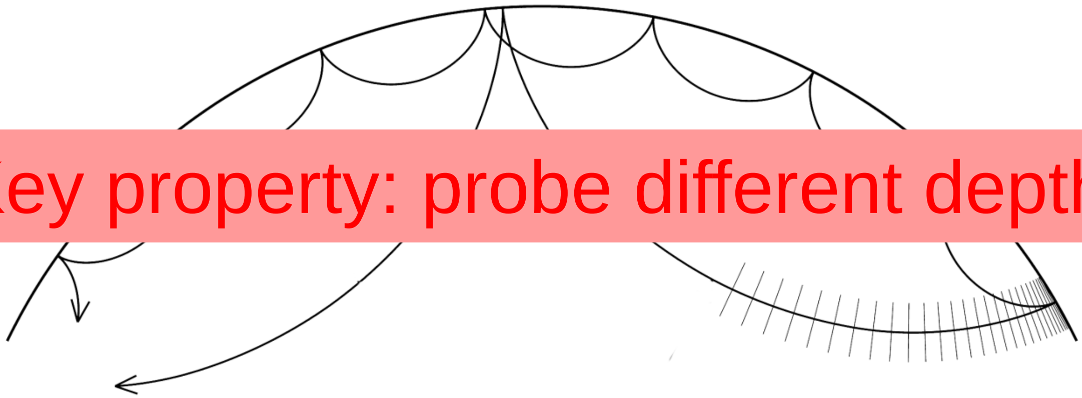
Trajectories of acoustic waves in interior



Key property: probe different depths

These more-deeply penetrating waves have longer horizontal wavelengths: bigger skip distance goes with larger λ_h [smaller k_h]

Courtesy J. Christensen-Dalsgaard

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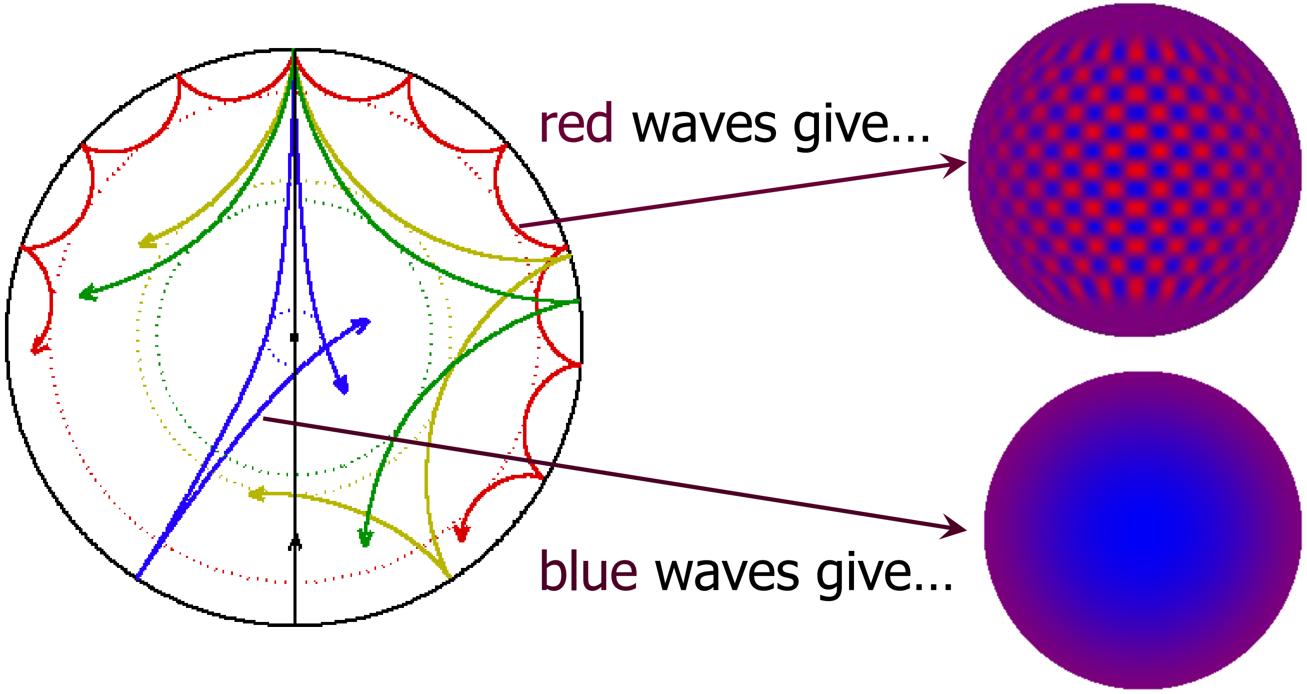
Standing acoustic wave patterns...

Internal acoustic ray paths

Surface displacement: oscillation patterns in 3D



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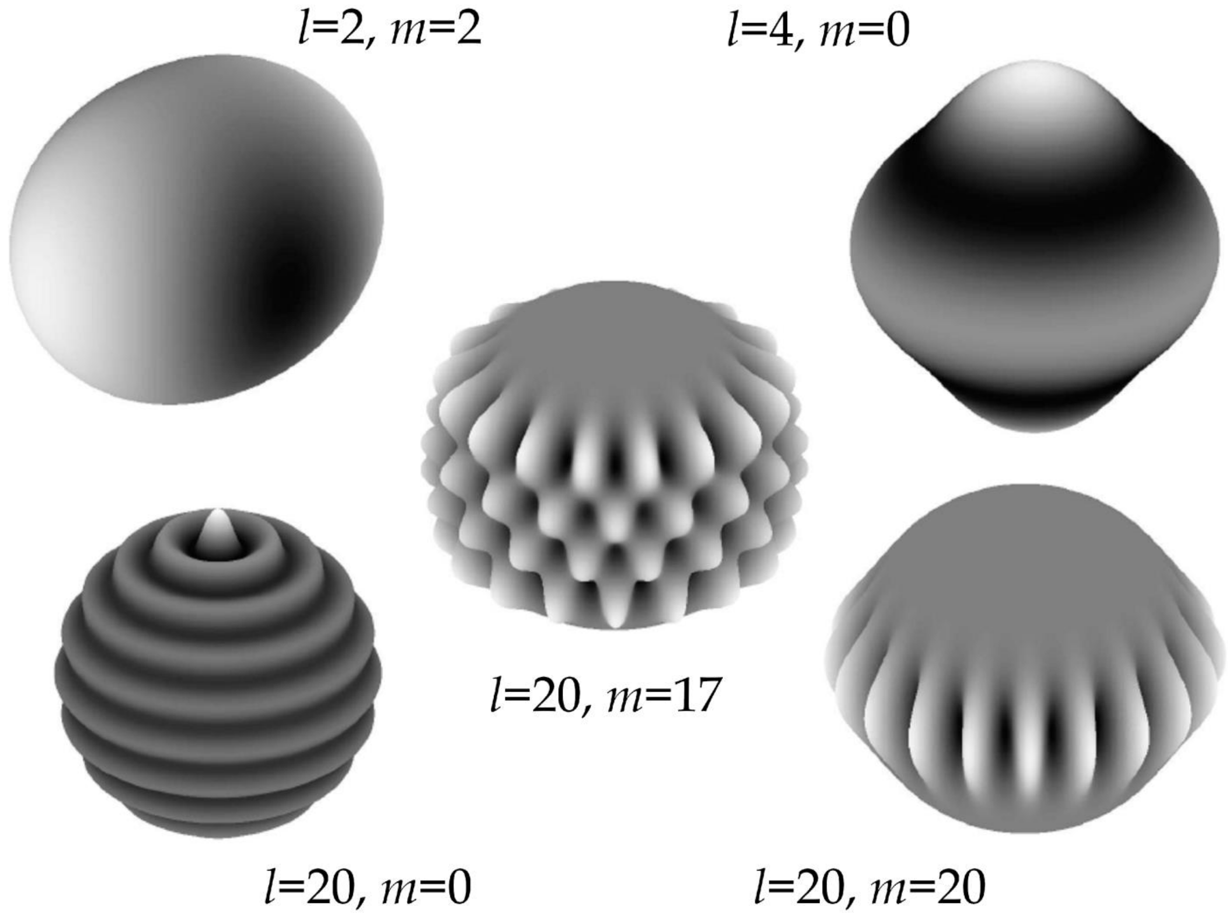
Categorization of modes

- Spatial part can be described by spherical harmonic functions
- Spherical harmonic integers, *l* and *m*, describe spatial pattern
- Radial order *n* corresponds to the number of nodes in the radial direction

Spherical Harmonics



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Categorization of modes

Angular degree, *l*, depends on horizontal wave number and outer radius of cavity according to:

$$k_h = \frac{2\pi}{\lambda_h} = \frac{L}{R} = \frac{\sqrt{l(l+1)}}{R}$$

So:

$$\lambda_h = \frac{2\pi}{k_h} = \frac{2\pi R}{L}$$

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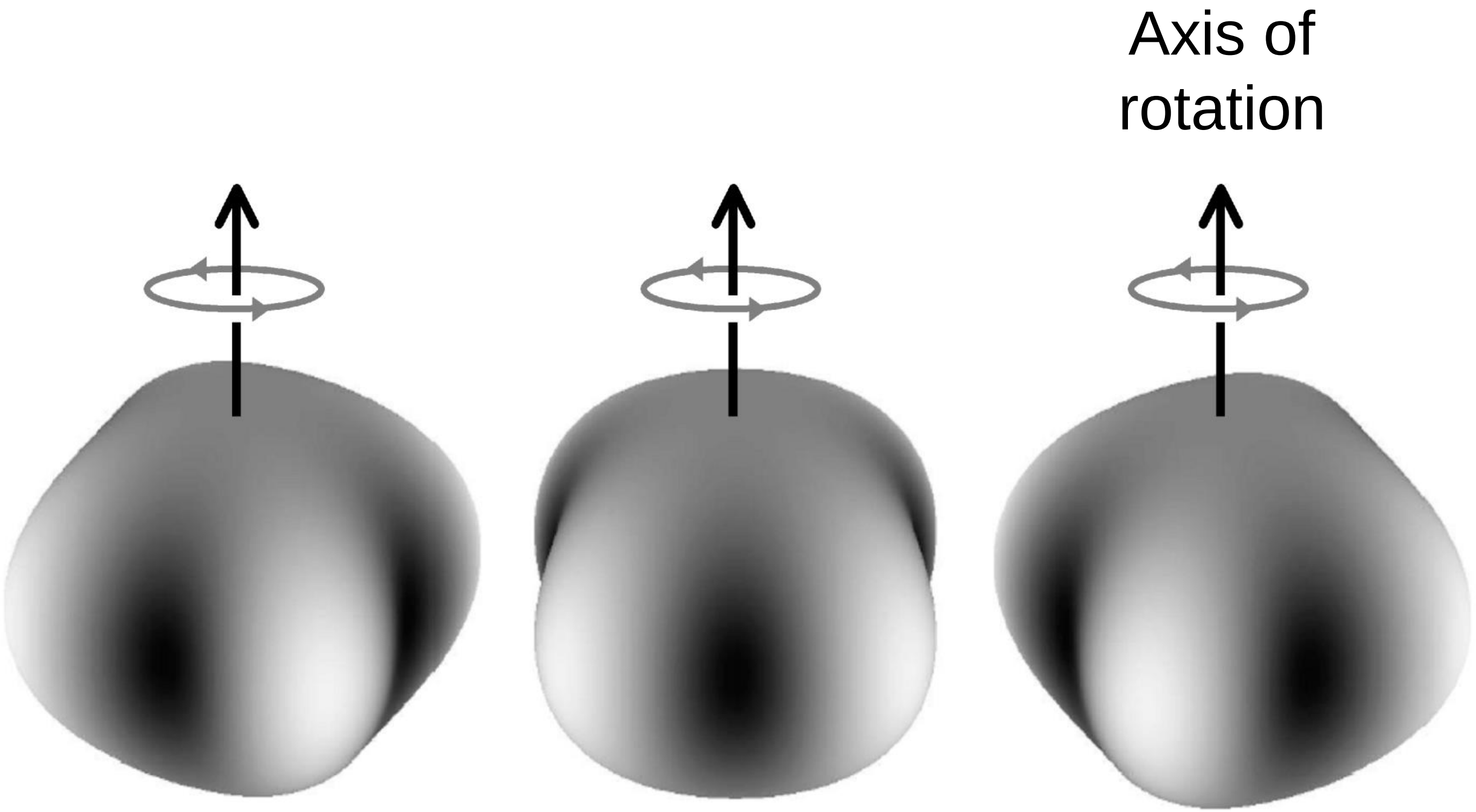
Rotational Splitting



Rotational Splitting

Shifted frequencies: $\omega_m \approx \omega_0 + m\Omega$

Ω is suitable average of position dependant angular velocity in cavity probed by mode

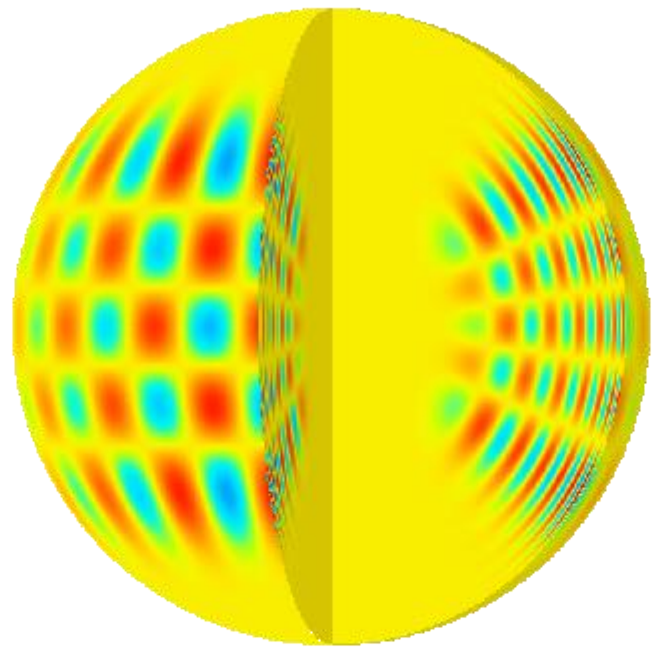
(Note: correction from coriolis force is small for Sun.)

Approximate magnitude of rotational splitting:

$$\Delta \nu_{\Delta|m|=1} = \frac{\Delta\omega_{\Delta|m|=1}}{2\pi} = \frac{\Omega}{2\pi} \approx 0.4\,\mu\text{Hz}$$

The Resonant Sun

Analogy of a musical instrument...



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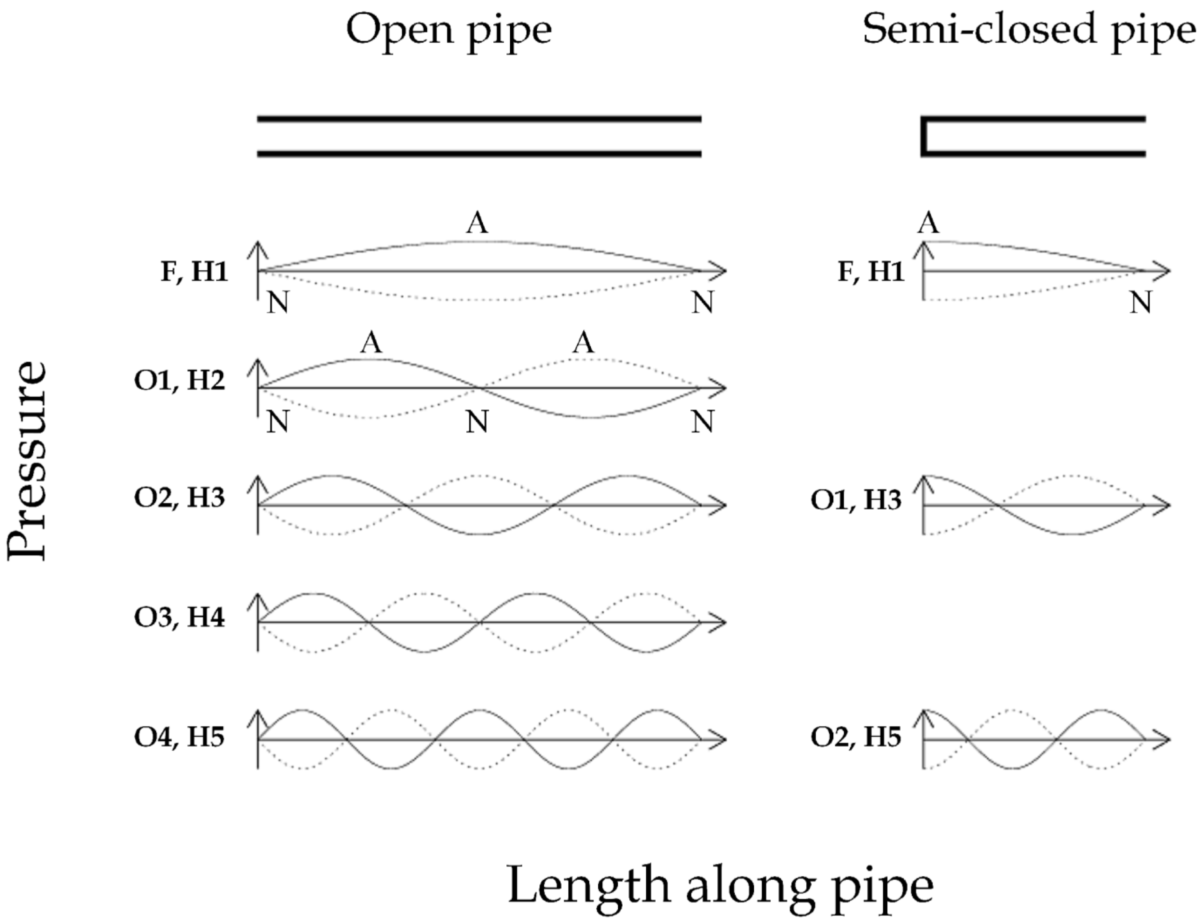
Condition for constructive interference

- Take a simple 1-D pipe
  - Pipe runs from $z=0$ to $z=L$
- Condition for standing waves depends on boundary conditions
  - *i.e.*, is pipe open or closed?

Resonance in simple 1-D pipes



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Fully open pipe

Fundamental (1st harmonic): $L = \frac{\lambda}{2}$

1st overtone (2nd harmonic): $L = \lambda$

2nd overtone (3rd harmonic): $L = 3\lambda / 2$

So:

$$L = \left(\frac{n+1}{2}\right)\lambda.$$

n=0, 1, 2… etc.

Fully open pipe

Then:

$$k_z L = \frac{2\pi}{\lambda}\left(\frac{n+1}{2}\right)\lambda,$$

where k_z is wave number:

$$\therefore\ k_z L = \int_L k_z dz = (n+1)\pi.$$

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Fully open pipe

- This is classic interference condition, *i.e.*,

$$\Delta\phi = \int_L k_z dz = (n+\alpha)\pi,$$

where α is a constant.

- Clearly α =1 for fully open pipe
- For semi-closed pipe, α =½

Interference condition in Sun

- We can write the same interference condition for waves trapped inside the Sun
- We have: integral of radial wave number between lower (r_t) and upper (R_t) turning points, *i.e.*,

$$\Delta\phi = \int_{r_t}^{R_t} k_r \, dr = (n+\alpha)\pi.$$

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Interference condition in Sun

- Value of α depends on boundary conditions at lower and upper turning points
- Need to consider each turning point separately, so two different contributions to α
- It turns out that $\alpha \approx 1.5$

But hang on! Sun is 3-D body

- First: generalise to resonance in 3-D pipe
  - Waves no longer plane
  - Allow component of motion at right angles to long axis of pipe: new families of modes
  - Solutions no longer sine waves: need Bessel functions!

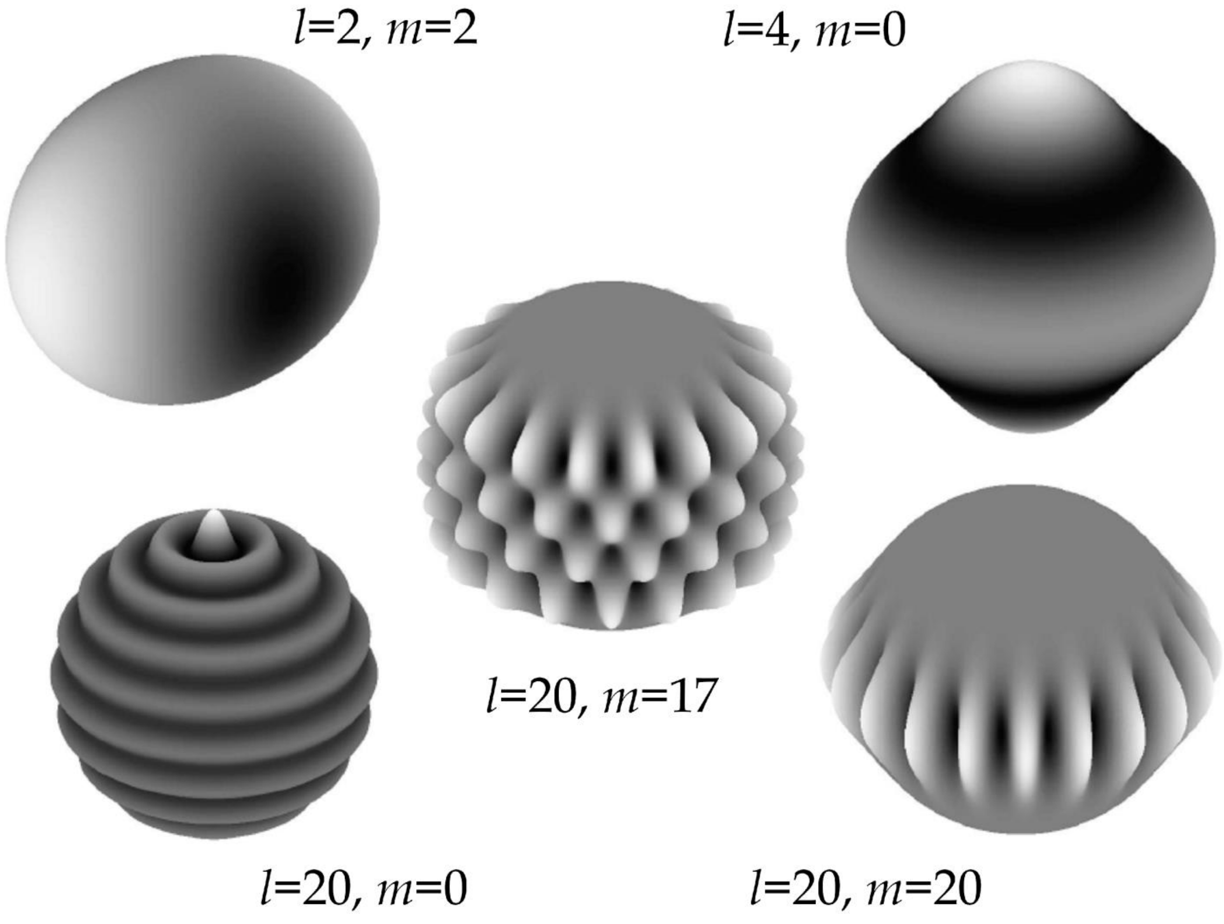
But hang on! Sun is 3-D body

- Make jump from cylindrical pipe to spherical geometry of Sun via cone (which can be treated as section of sphere)
- Again, two-part solution:
  - Radial part
  - Transverse part (spherical harmonics)

But hang on! Sun is 3-D body

- For cone, radial part dominates
  - Transverse modes can become more important for geometry of flaring bell
- For sphere, with restriction of narrow bore removed, get rich spectrum of transverse modes

Spherical Harmonics



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The frequency spectrum

Start with example of fully open 1-D pipe

Fundamental (first harmonic) frequency:

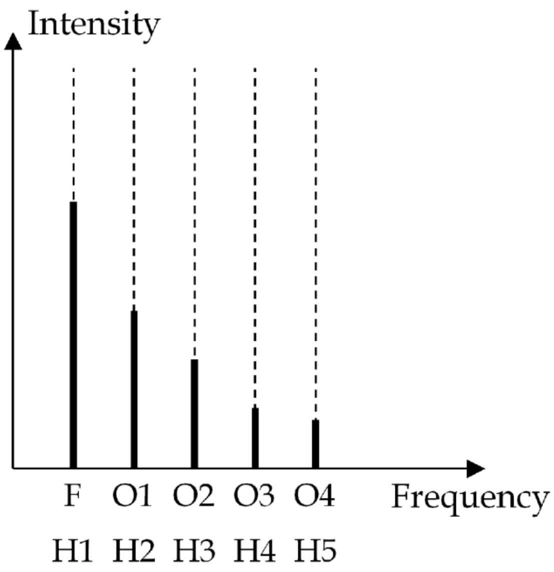
$$\nu_F = \nu_{H1} = \frac{c}{2L}.$$

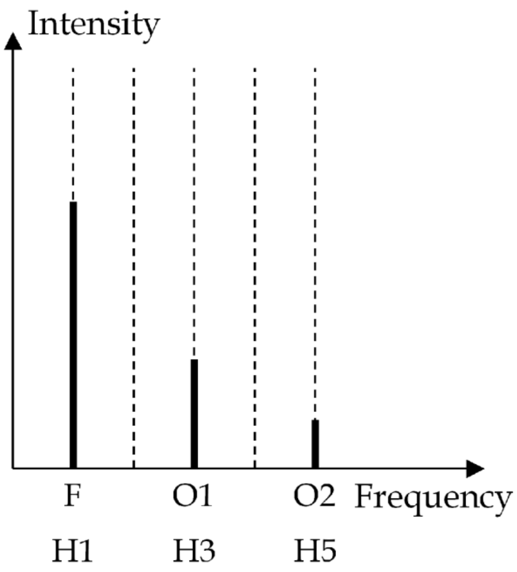
Overtone (higher harmonic) frequencies:

$$\nu_{O1} = \nu_{H2} = \frac{2c}{2L},$$

$$\nu_{O2} = \nu_{H3} = \frac{3c}{2L}$$ … and so on

The frequency spectrum





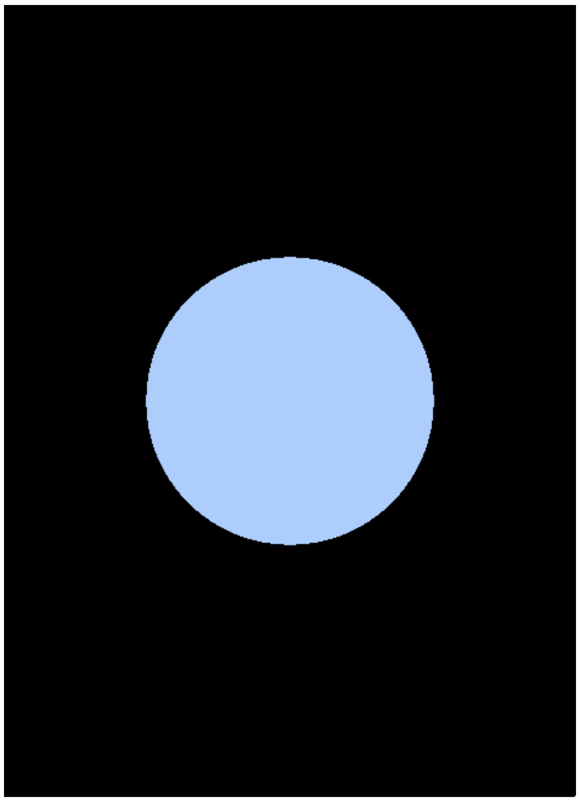
Fully open
length *L*

Semi-closed
length *L*/2

The frequency spectrum

- In 3-D case, each transverse solution has its own set of overtones
- So, for Sun (with spherical harmonic solutions giving transverse part):
  - each angular degree, *l*, has its own family of overtones (described by order *n*)

Pulsation Timescale



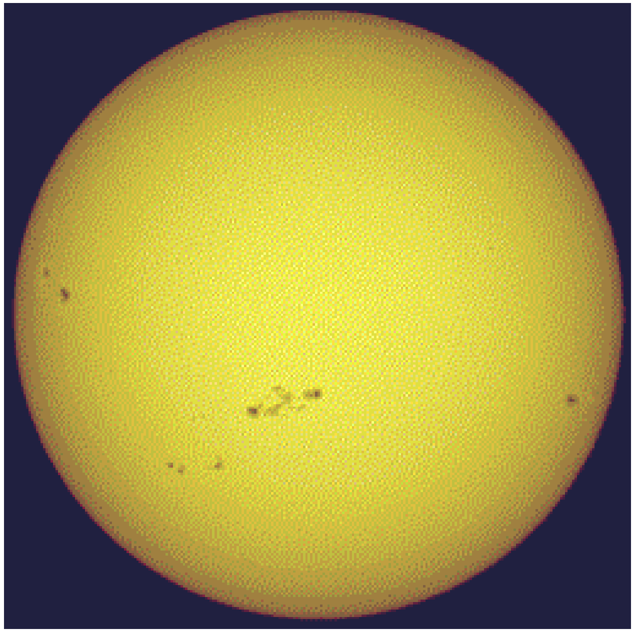
- Fundamental period of radial pulsation:

$$\Pi \propto \langle \rho \rangle^{-1/2}$$

Ritter 1880; Shapley, 1914

- Estimate period from sound crossing time
- Period similar to dynamical timescale ('free fall' time)

Pulsation Timescale



Courtesy D. Hathaway

Sun: fundamental radial mode period

$$\Pi_f \approx 1 \text{ hour}$$

$$\nu_f \approx 250 \ \mu\text{Hz}$$

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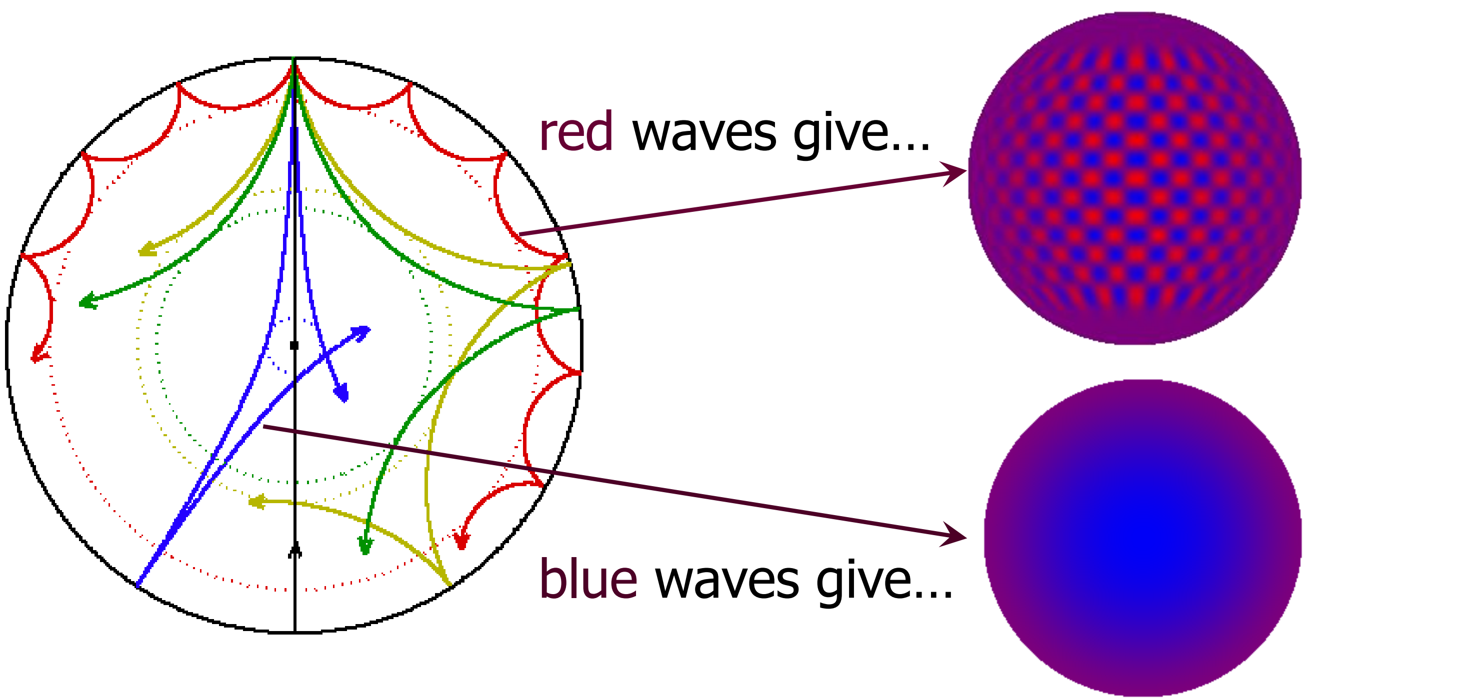
Standing acoustic wave patterns...

Internal acoustic ray paths

Surface displacement: oscillation patterns in 3D



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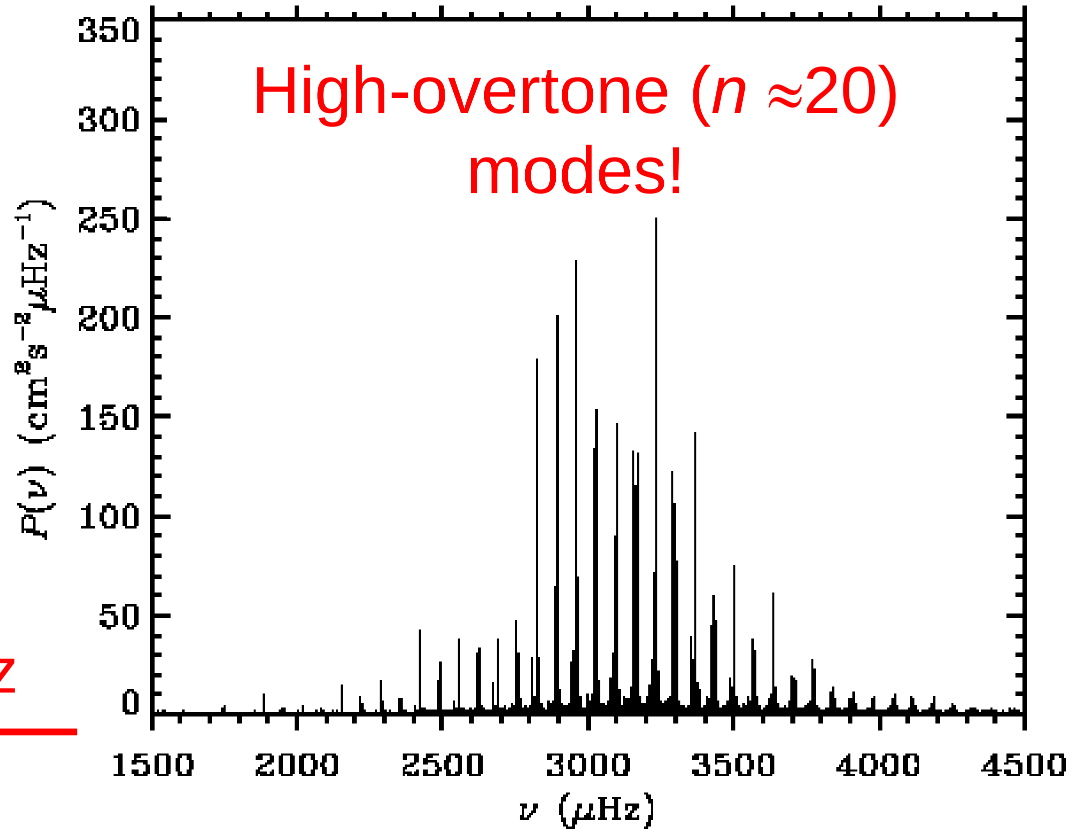
Frequency spectrum of low-degree (low-l) modes (contains overtones of $0 \leq l \leq 3$)



250 μHz

BiSON data

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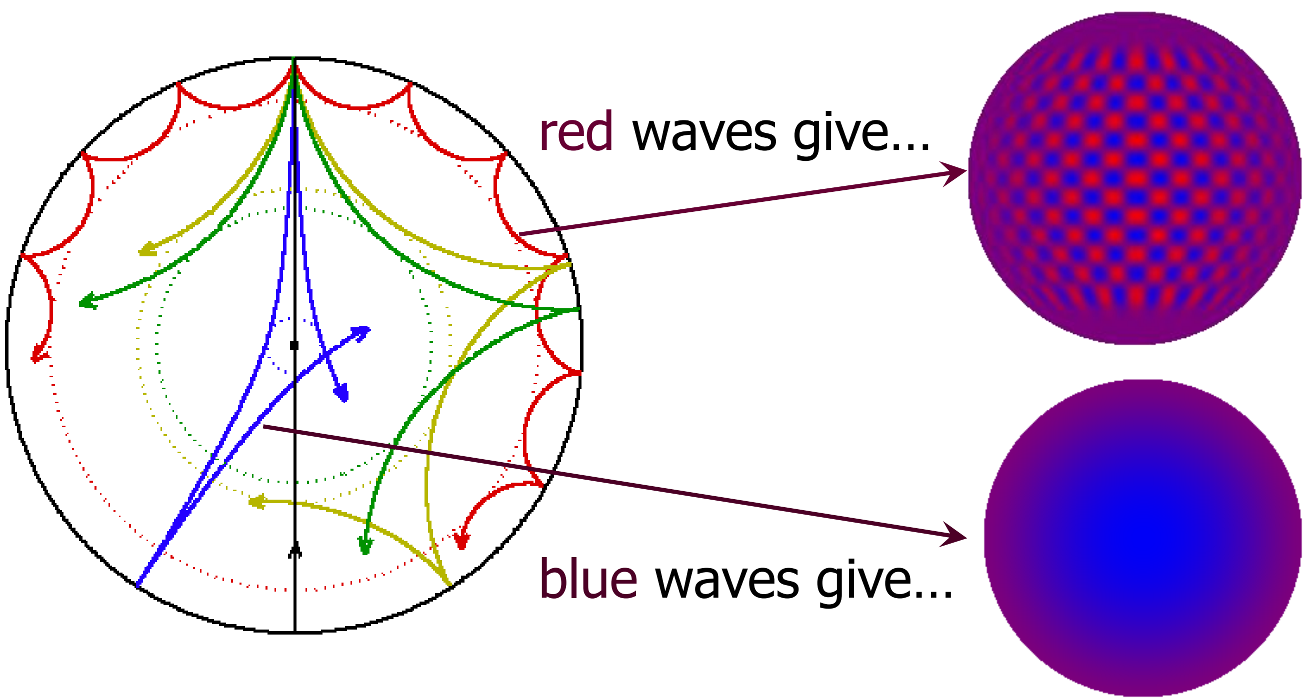
Standing acoustic wave patterns...

Internal acoustic ray paths

Surface displacement: oscillation patterns in 3D



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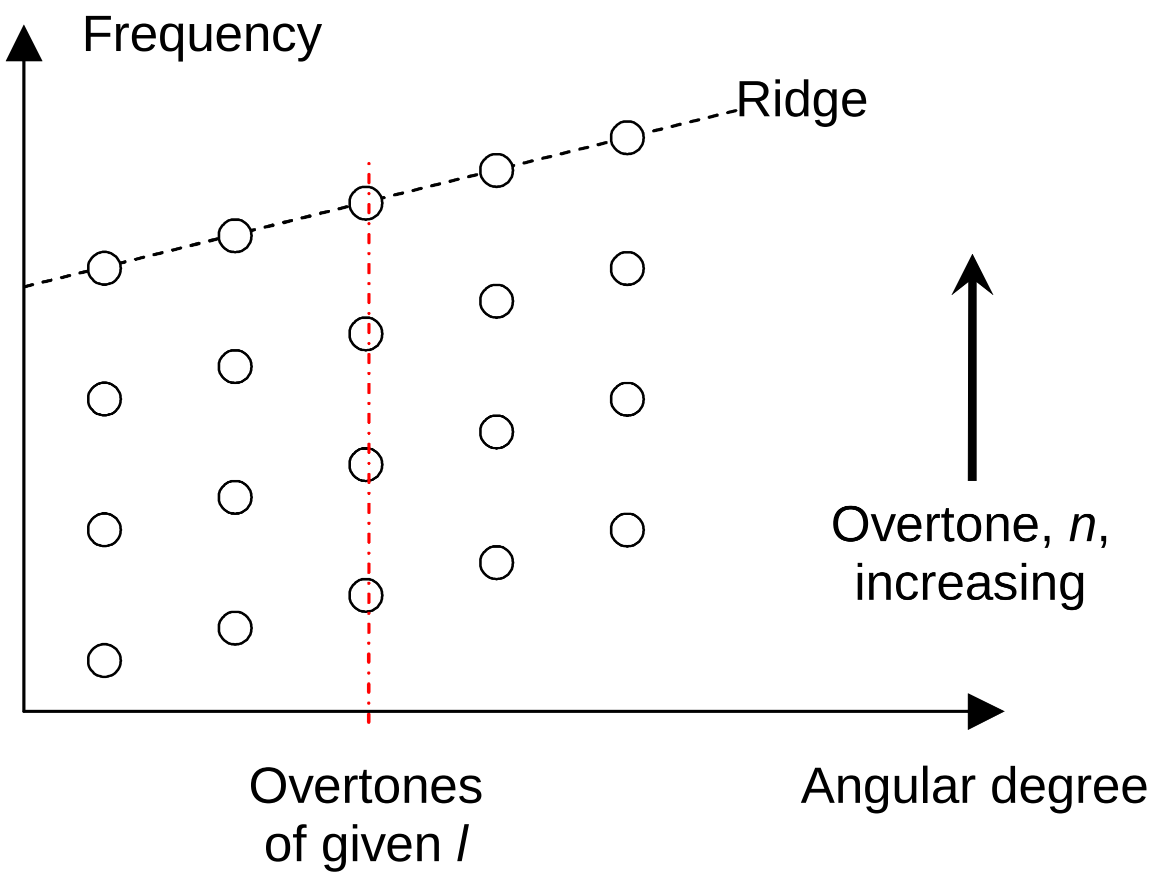
Frequency spectrum: *l*-ν diagram

Consider the overtones of each degree, *l*



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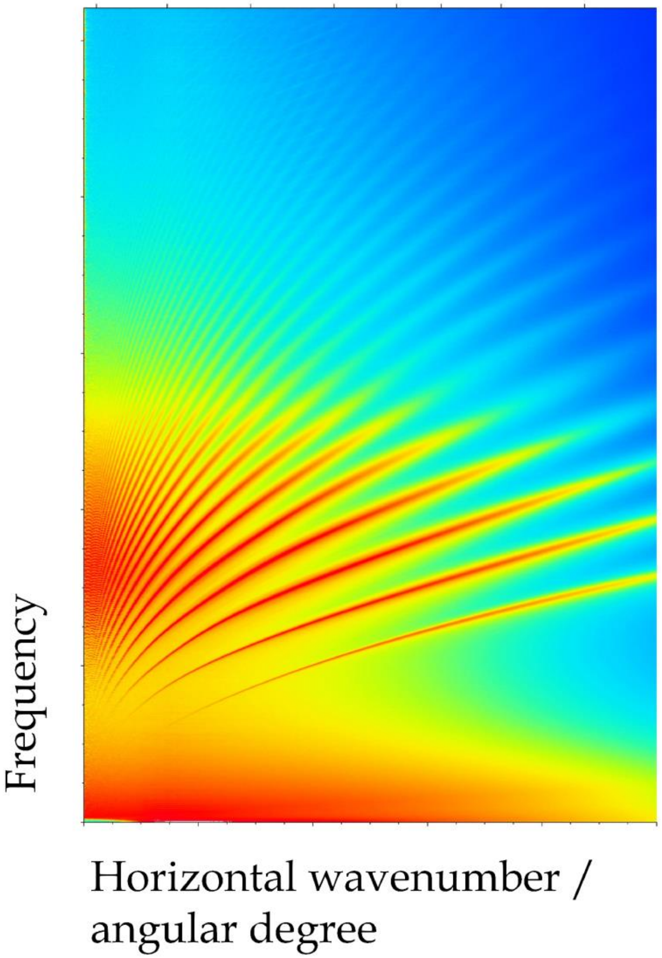
Frequency spectrum: *l*-ν diagram



Data collected by MDI instrument on board SOHO

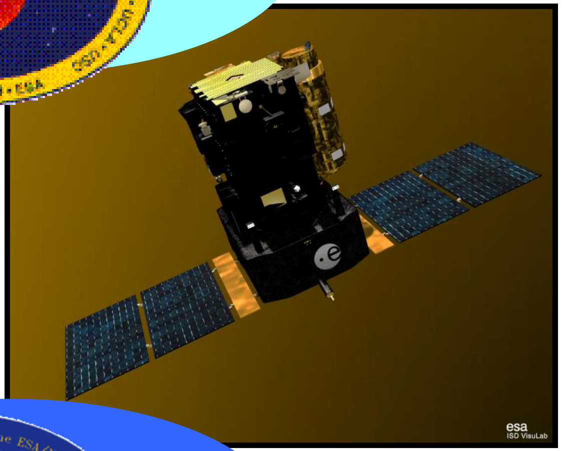
Sun-as-a-star observations

BiSON
6 stations

VIRGO/SPM

Resolved-Sun Observations

SOI/MDI

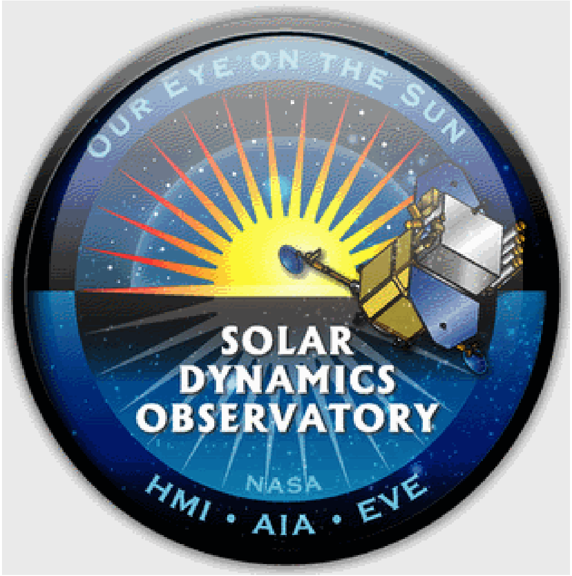


VIRGO/LOI

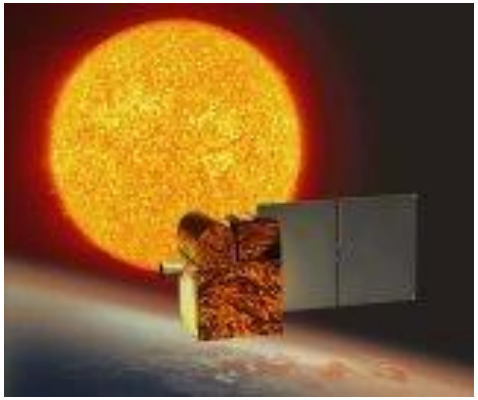
GONG 6 stations

Resolved-Sun Observations

HINODE



HMI/SDO



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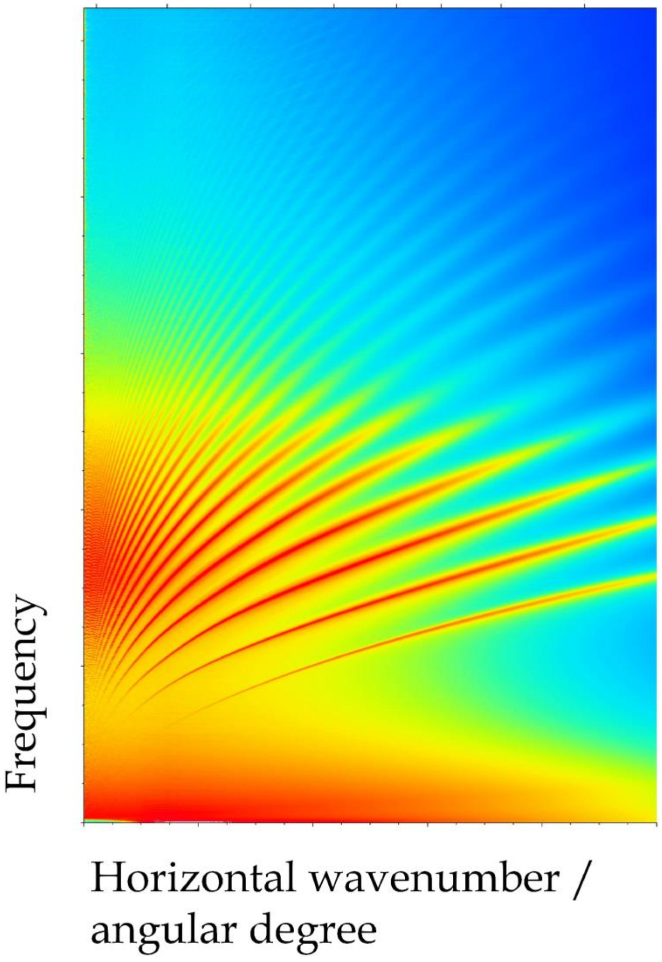
Global and local helioseismology

- Global seismology:
  - Constituent waves live long enough to travel round the Sun
  - Modes give longitudinal average of properties (also cannot distinguish asymmetry in properties above and below equator)

Global and local helioseismology

- Local seismology:
  - Do not wait for resonance to establish globally
  - Observe effects of interference in local volumes beneath surface

Frequency spectrum: *l*-ν diagram



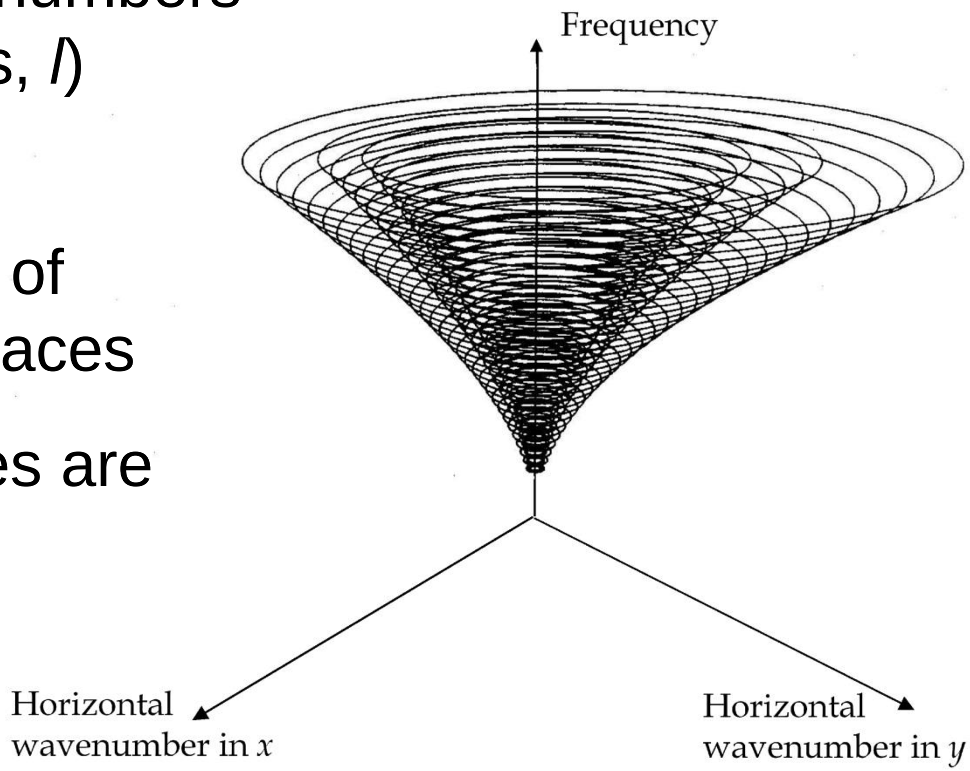
Data collected by MDI instrument on board SOHO

Local methods: rings and trumpets

Resolve into orthogonal horizontal wave numbers (angular degrees, *l*)

2-D *l*-ν diagram becomes series of nested 3-D surfaces

What were ridges are now flaring 'trumpets')



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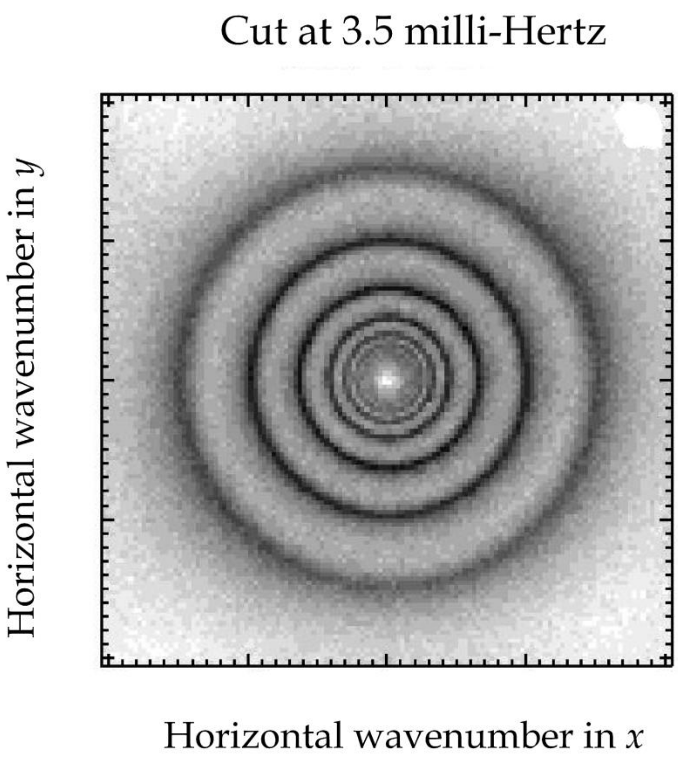
Local methods: rings and trumpets

Take cut at fixed frequency: get series of rings

Analysis of rings can be used to measure flows, fields *etc*., which distort shapes of rings

Measure properties beneath small patches on surfaces, *e.g*., beneath active regions



Courtesy D. A. Haber and collaborators

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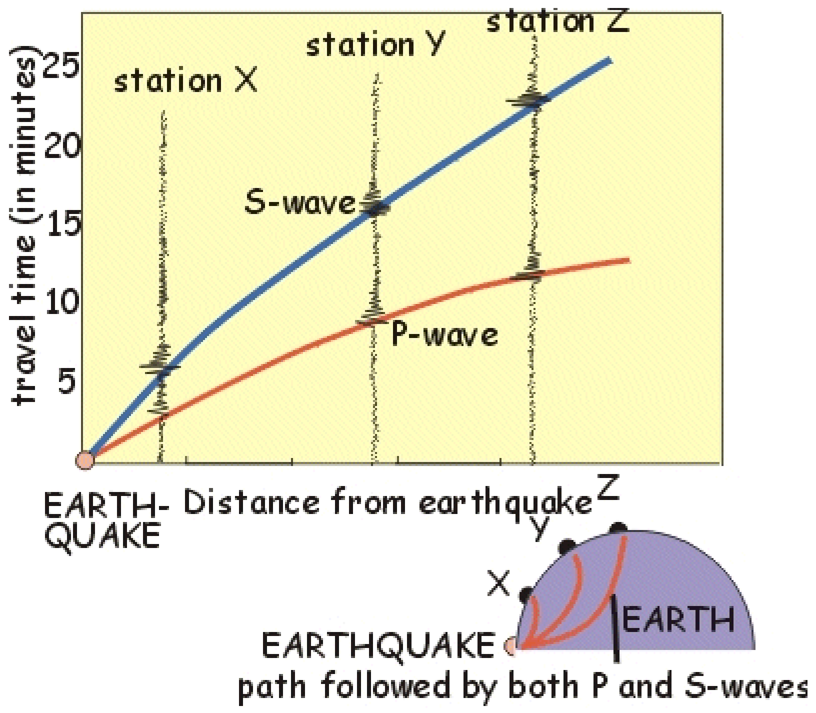
Local methods: time-distance helioseismology

Analogue of terrestrial time-distance methods

Measure time taken for waves to reach detectors from natural, or man-made, seismic events

Use this information to infer internal properties

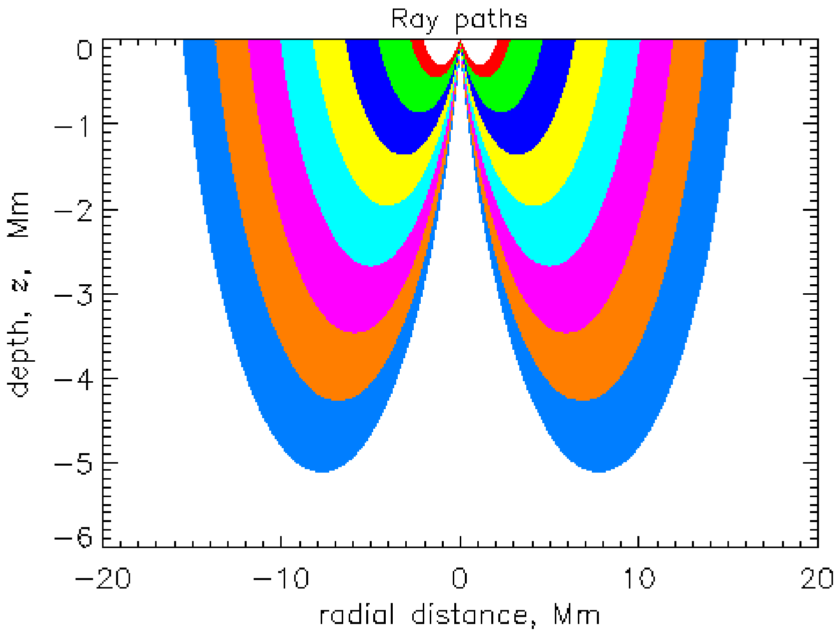


Local methods: time-distance helioseismology

Can try something similar on Sun, but:

•We cannot create our own seismic events

•Seismic generation of waves takes place at multitudinous locations across surface of Sun!



Courtesy A. G. Kosovichev, SOI Stanford

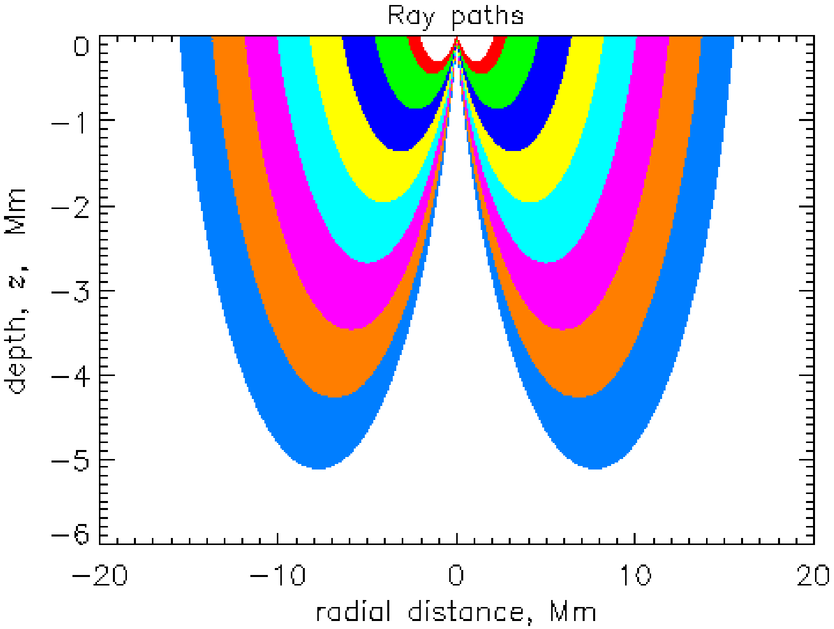
Local methods: time-distance helioseismology

- Is there a way round the problem of having many sources?
- Yes: use cross correlation techniques

Local methods: time-distance helioseismology

To first order, waves launched at given angle take same time to reappear at surface

This is so-called single-skip time



Courtesy A. G. Kosovichev, SOI Stanford

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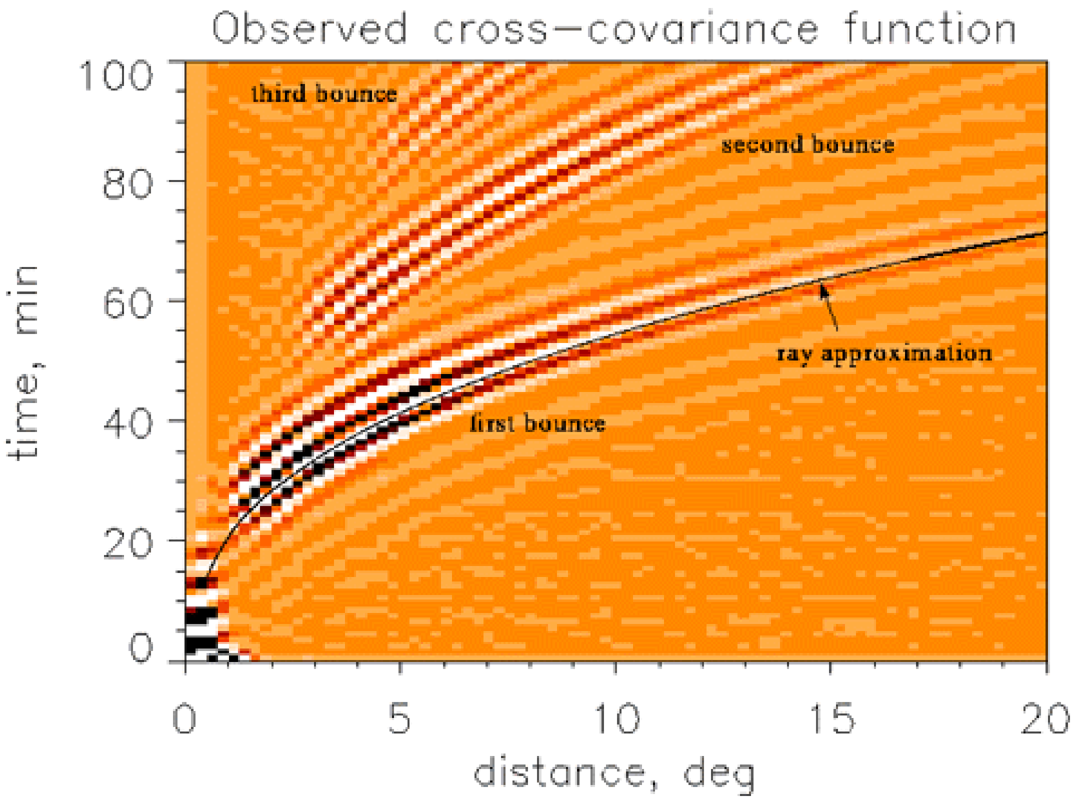
Local methods: time-distance helioseismology

- Take two locations on surface
- Measure Doppler velocity or intensity at these locations
  - Separation will correspond to skip distance for waves launched *at particular angle*
  - Signals will be strongly correlated at separation in time corresponding to time to make single skip
  - Can also pick up time to make two skips… and three

Local methods: time-distance helioseismology



Courtesy A. G. Kosovichev, SOI Stanford

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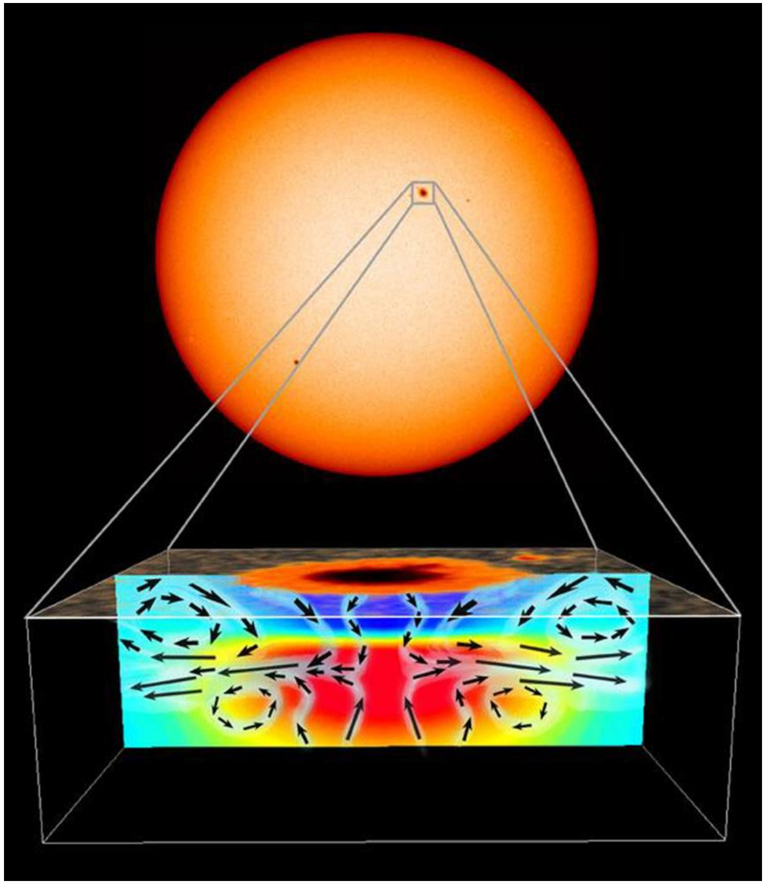
Local methods: time-distance helioseismology

- In practice, cross correlated between central patch and surrounding annulus
- Search for strong correlations at each separation
- Can build solar equivalent of terrestrial time-distance plot
- Infer internal properties from travel-time information

Flows and wave speed variation beneath sunspot (from local methods)



Arrows show flows:

Larger →

Smaller →

Colours show wave-speed:

Faster… in **red**

Slower… in **blue**

Courtesy A. G. Kosovichev, SOI Stanford

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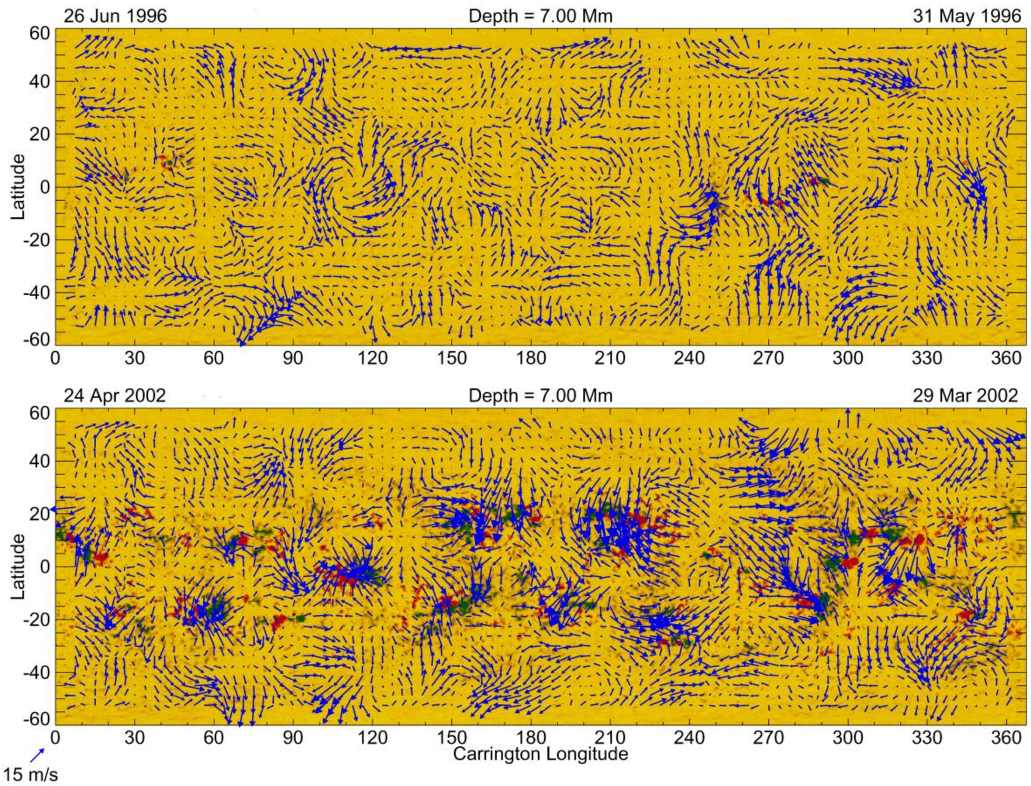
Solar Sub-Surface Weather (Local methods)

Flows (arrows) beneath regions of magnetic flux (red)



Measure flows underneath small patches

Rotation brings new patches into view

Build up strips, side-by-side, in longitude

Courtesy D. A. Haber and collaborators

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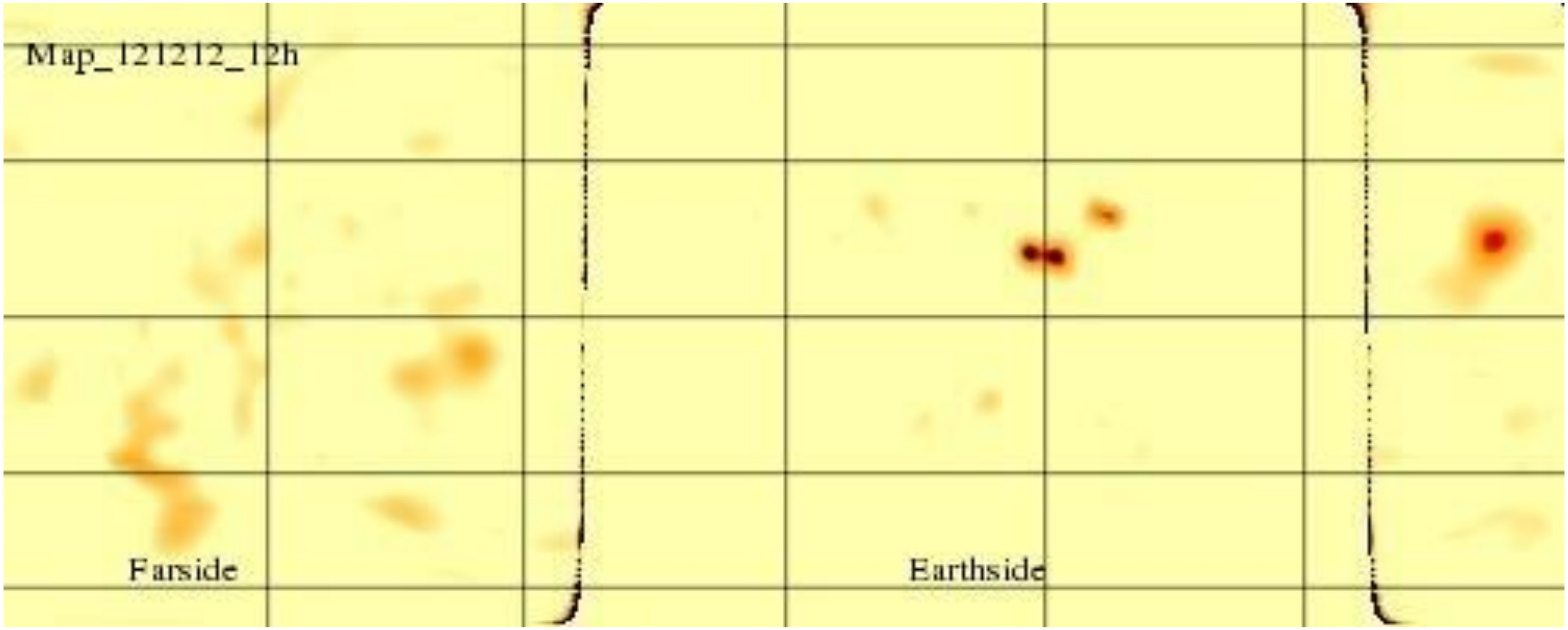
Space weather predictions

Far-side imaging of active regions



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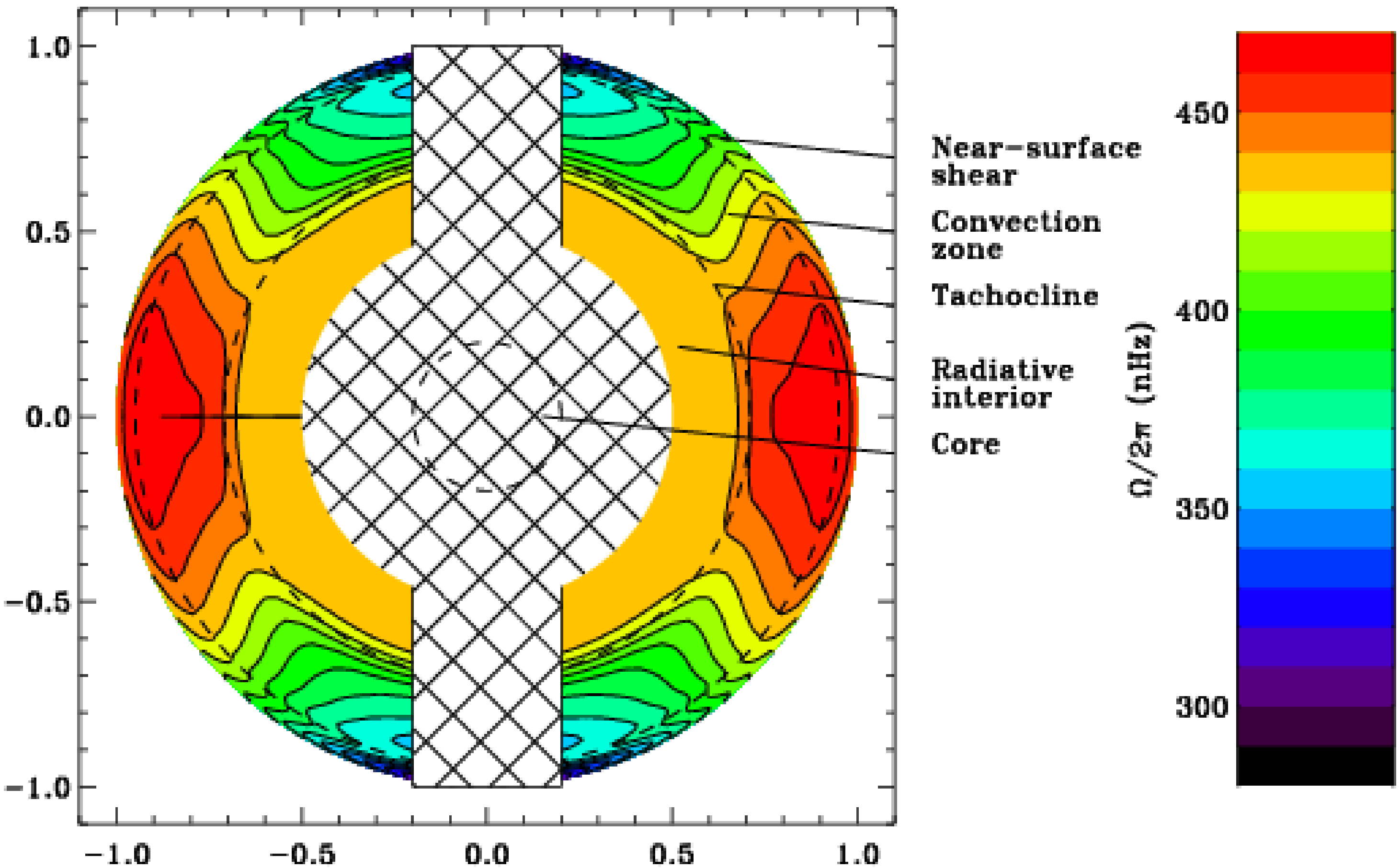
Internal Solar Rotation



GONG data

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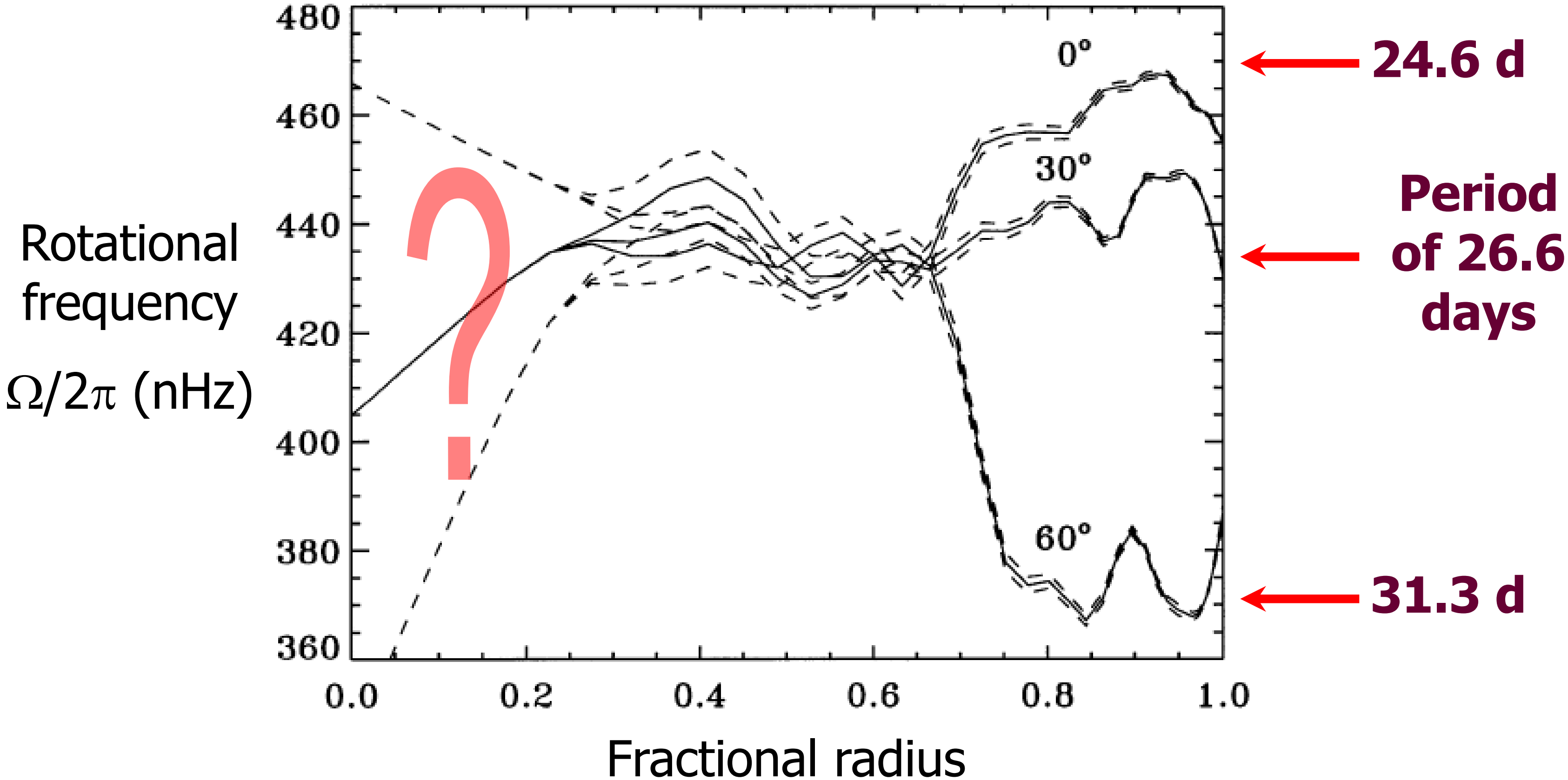
Slow Rotation of the Deep Interior!



Chaplin et al., 1999, MNRAS

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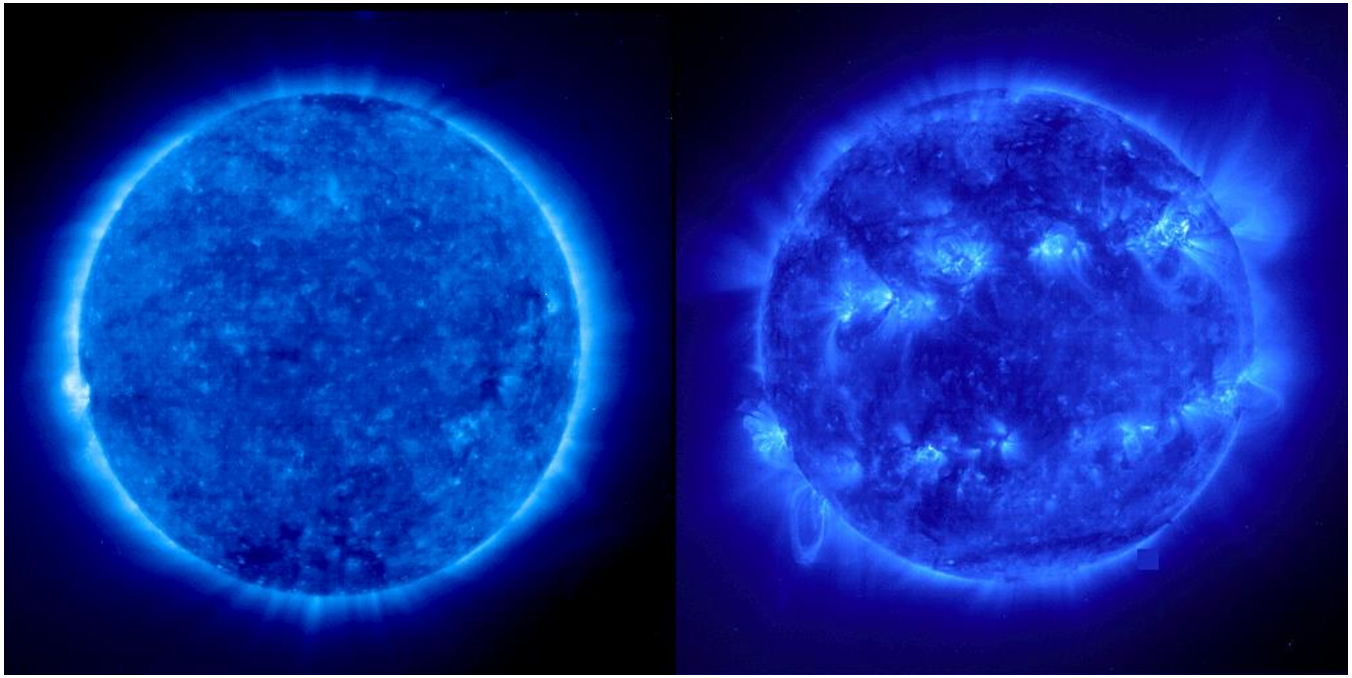
Solar Activity and the Solar Cycle



Courtesy of SOHO ESA/NASA

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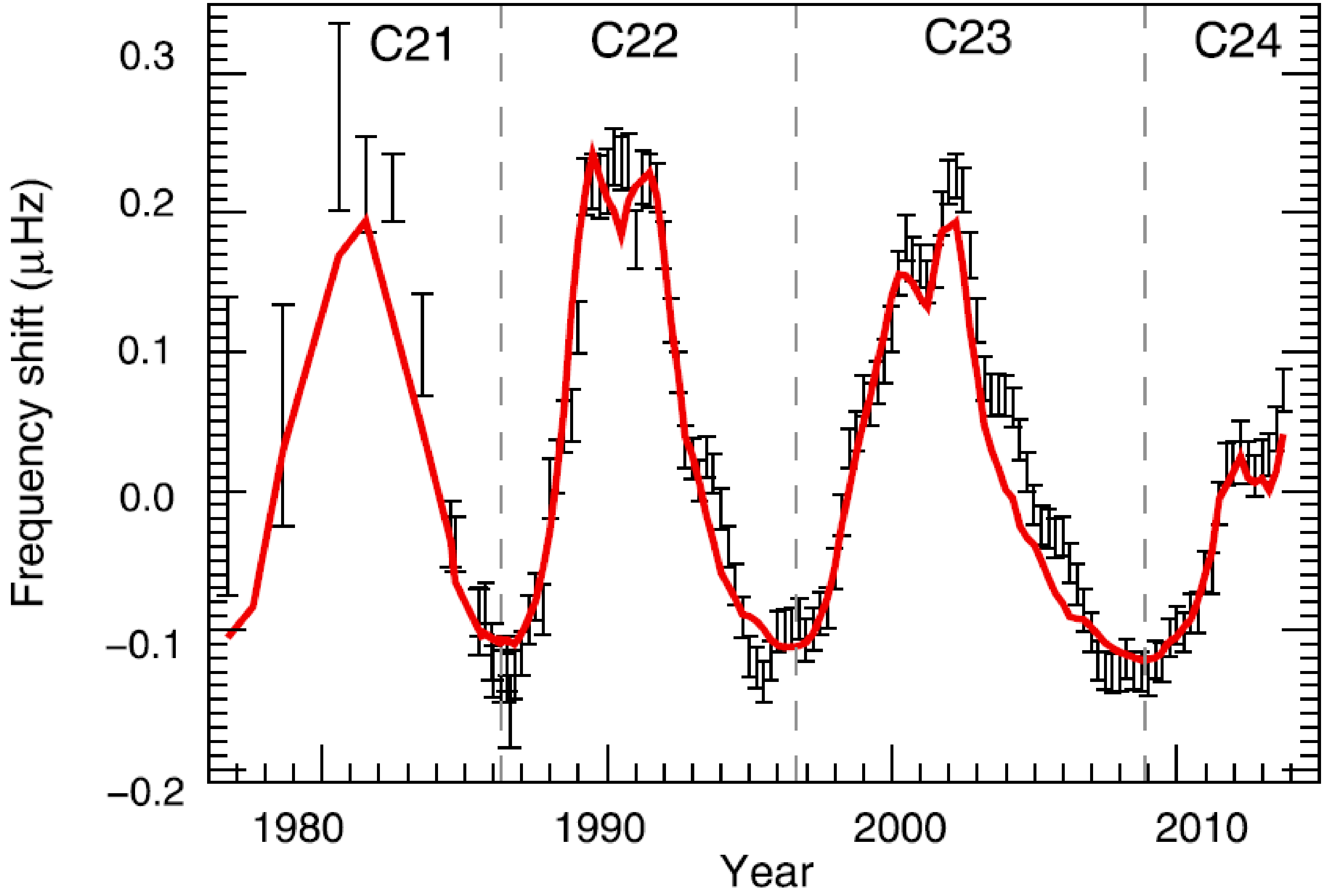
“Sounding” stellar activity cycles: Sun

BiSON Sun-as-a-star data



scaled 10.7-cm radio flux

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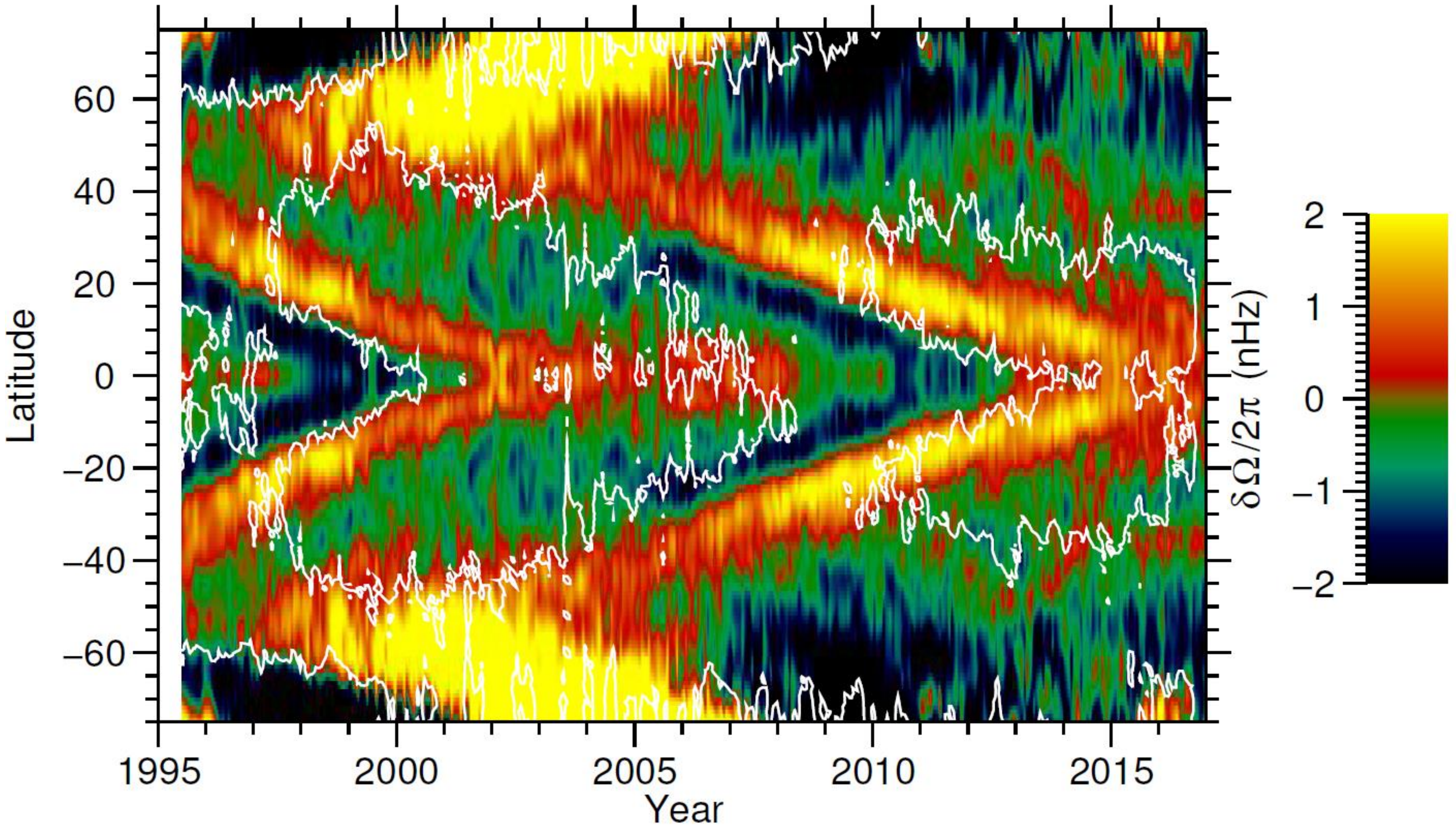
Near-surface flows



Howe et al., in preparation

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End